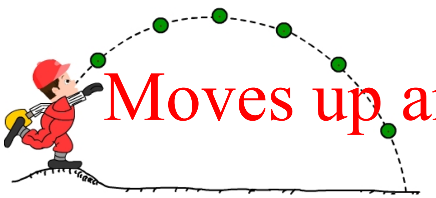
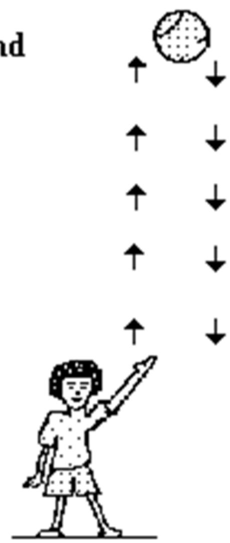
Things do not always move only along only one axis. So, we'll have to learn some small tweaks to our problem solving strategies to allow us to apply the same ideas that we've already learned.





The main difference is we will have vectors.

Today's Focus: Vectors

Next Class: Using Them

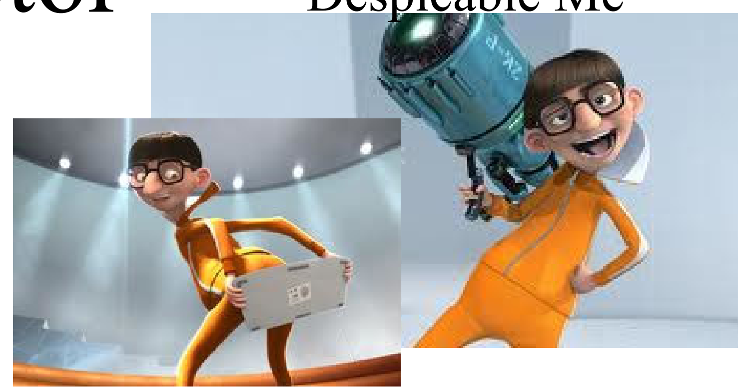
After today, you should be able to:

- Understand vector notation
- Use basic trigonometry in order to find the x and y components of a vector

  (only right triangles)

- Add and subtract vectors

Villian from movie Despicable Me



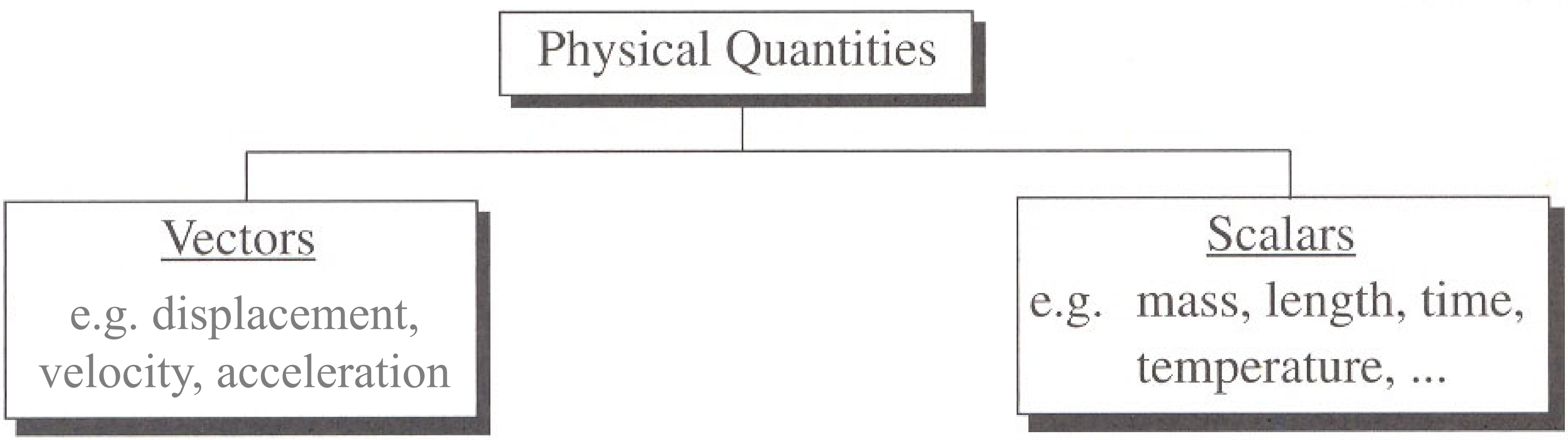
Practice Problems: trig practice (1.45, 1.47, 1.49, 1.51, 1.53), vectors (1.55, 1.57, 1.61, 1.63, 1.65, 1.67, Conceptual problem 1.15)

Quick Review

Quantities that are determined by a magnitude alone are called *scalars*.

Quantities that have both magnitude and direction are called *vectors*.

In conclusion, physical quantities can be classified into two types:



Vector Notation Varies

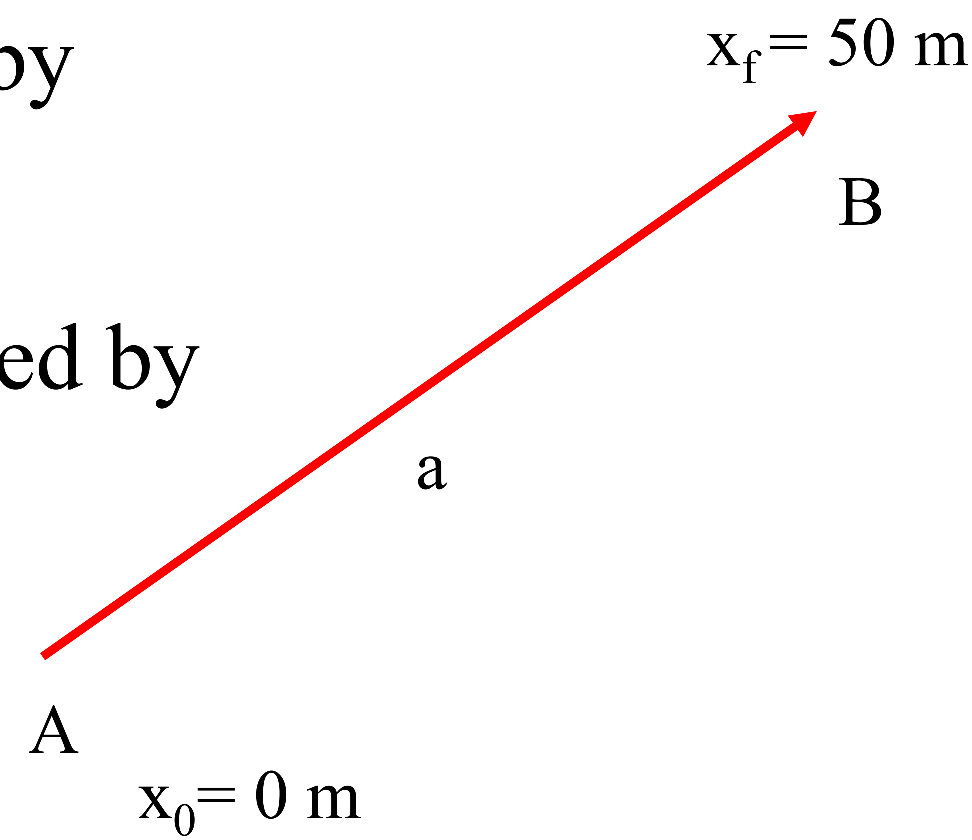
A vector may be represented by an arrow whose direction represents the direction of the vector and whose length represents the magnitude.

The vector may be called by

$\overrightarrow{AB}$, $\vec{a}$, $\underline{a}$, $\boldsymbol{a}$ *(bold)*

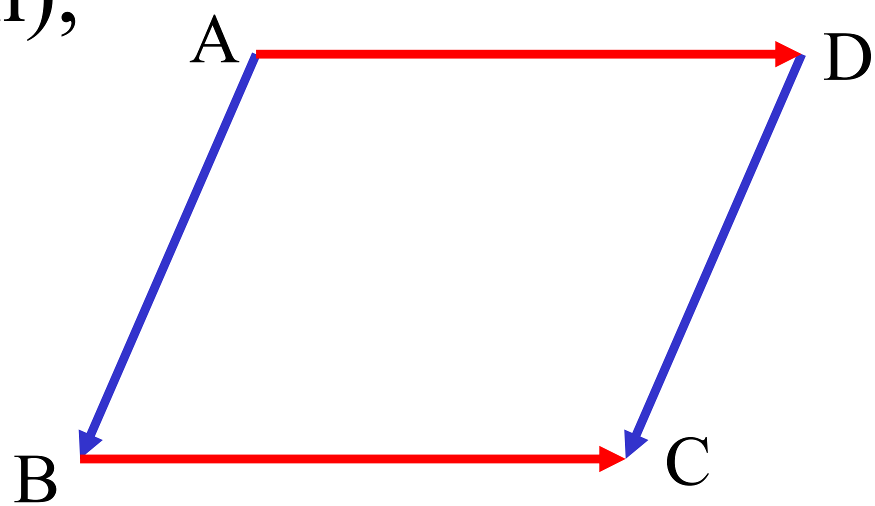
and its magnitude is denoted by

$\left|\overrightarrow{AB}\right|$, $\left|\vec{a}\right|$, $|\underline{a}|$, $|\boldsymbol{a}|$, a



Two vectors are said to be *equal* ONLY if they have the same magnitude AND direction.

ABCD is a parallelogram (defined by a shape with 4 sides, where all opposite sides have equal length),



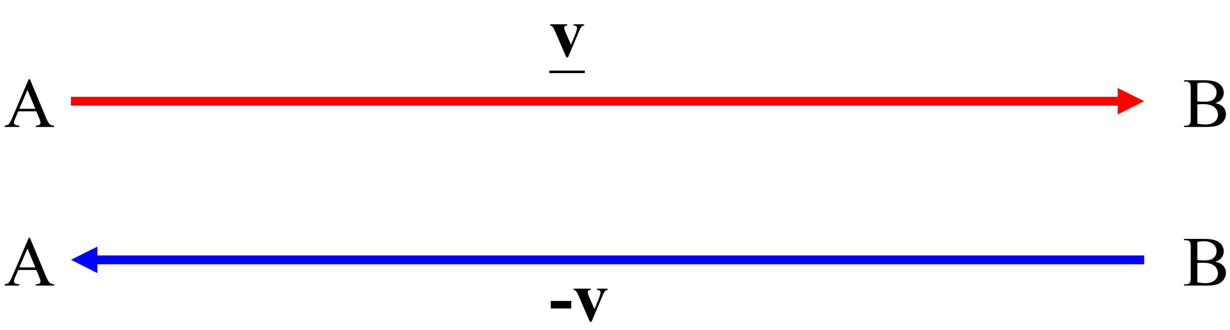
then $\overrightarrow{AB} = \overrightarrow{DC}$

and $\overrightarrow{AD} = \overrightarrow{BC}$

How about $\overrightarrow{AB}$ and $\overrightarrow{CD}$?

The negative vector of $\underline{\mathbf{v}}$, denoted by $-\underline{\mathbf{v}}$, is a vector having equal magnitude but opposite direction to $\underline{\mathbf{v}}$. Therefore

$$\overrightarrow{AB} = -\overrightarrow{BA}$$



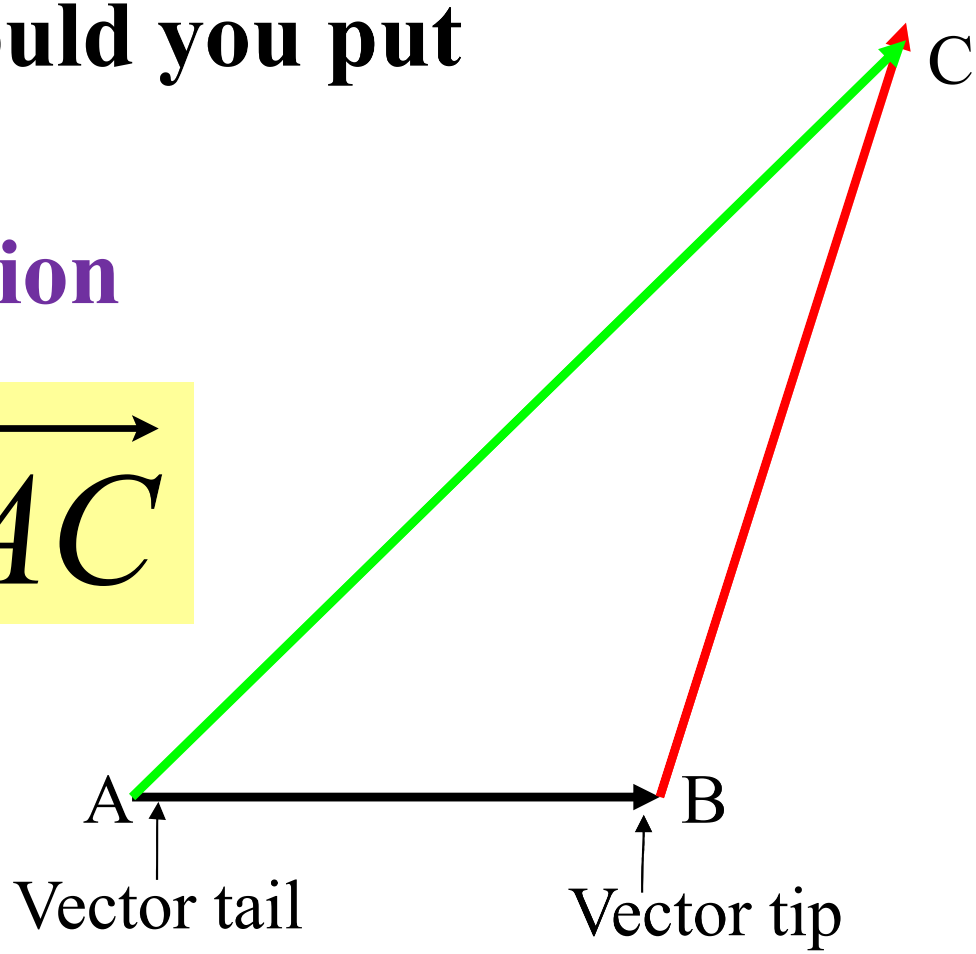
Basic Operation of Vectors: Addition

Q16

If you were to add the red vector to AB using the triangle law of addition, where would you put it?

triangle law of addition

$$\overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AC}$$



a) **With tail starting at A**
b) **With tail starting at B**
c) **Doesn't matter. In the triangle law, you would get the same answer by closing the triangle either way.**

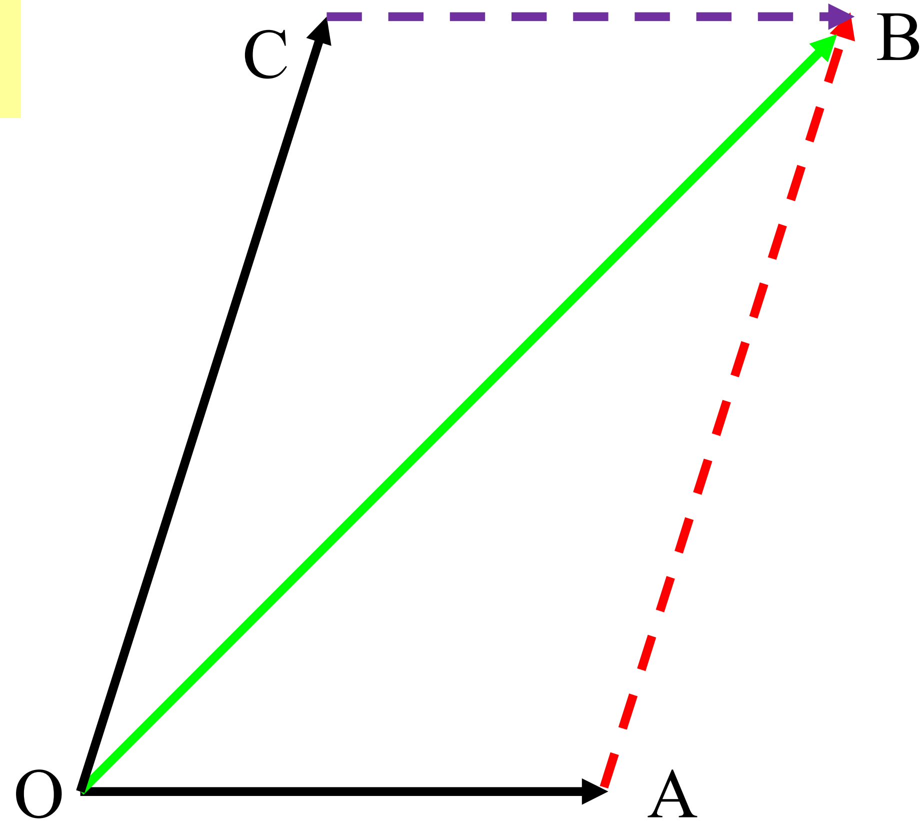
Basic Operation of Vectors

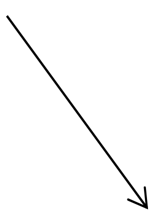
Q17

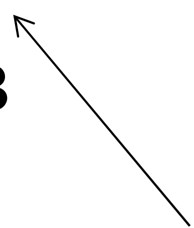
parallelogram law of addition

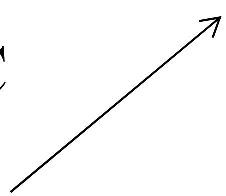
$$\overrightarrow{OA} + \overrightarrow{OC} = \overrightarrow{OB}$$

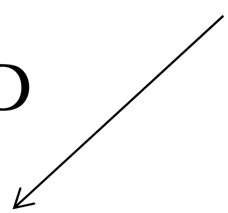
If you were to add these two vectors, roughly what direction would your result point?



A 

B 

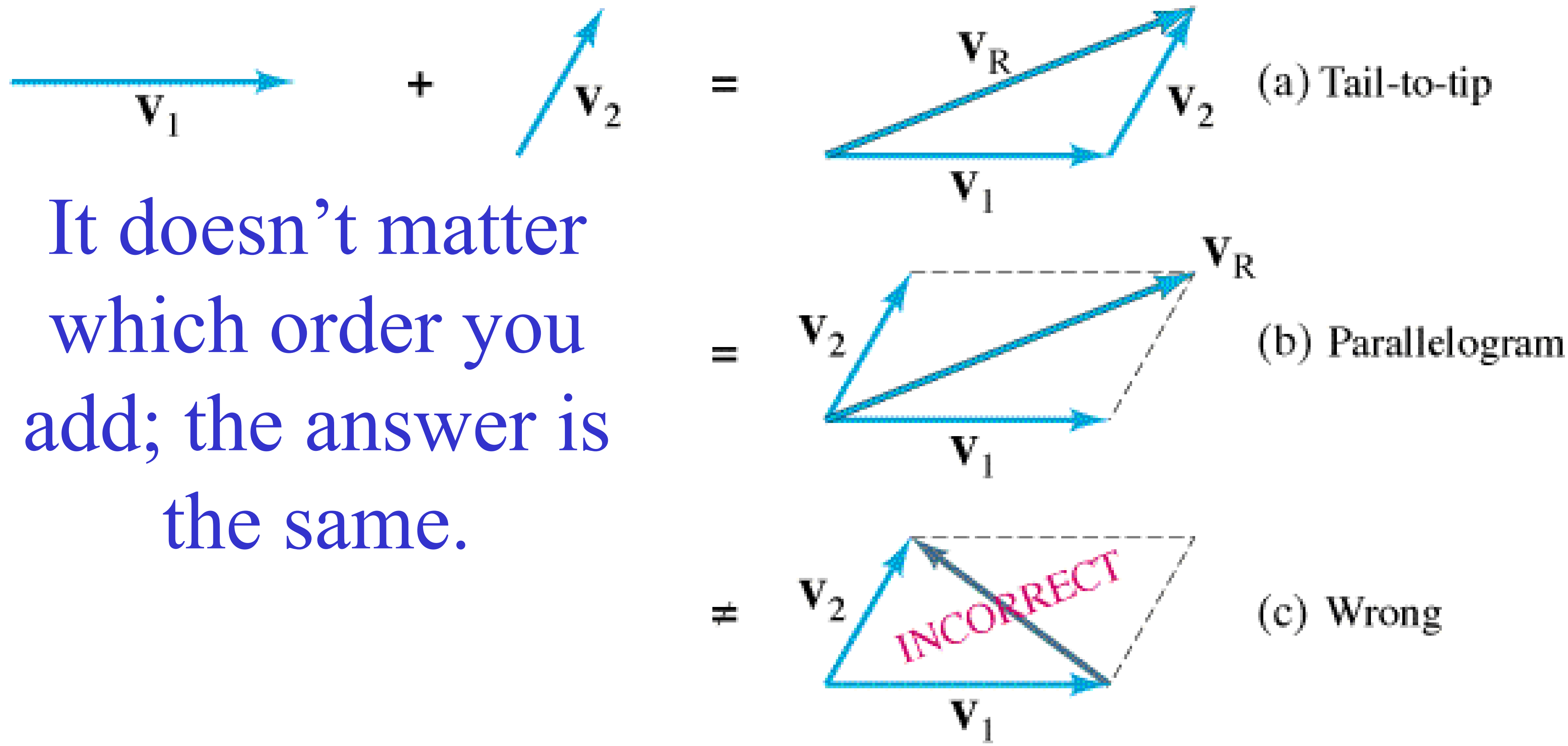
C 

D 

E None of the above

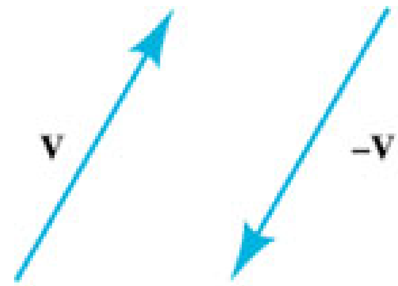
Vector Arithmetic

Addition: $\vec{V}_1 + \vec{V}_2 = \vec{V}_{Result}$



It doesn't matter which order you add; the answer is the same.

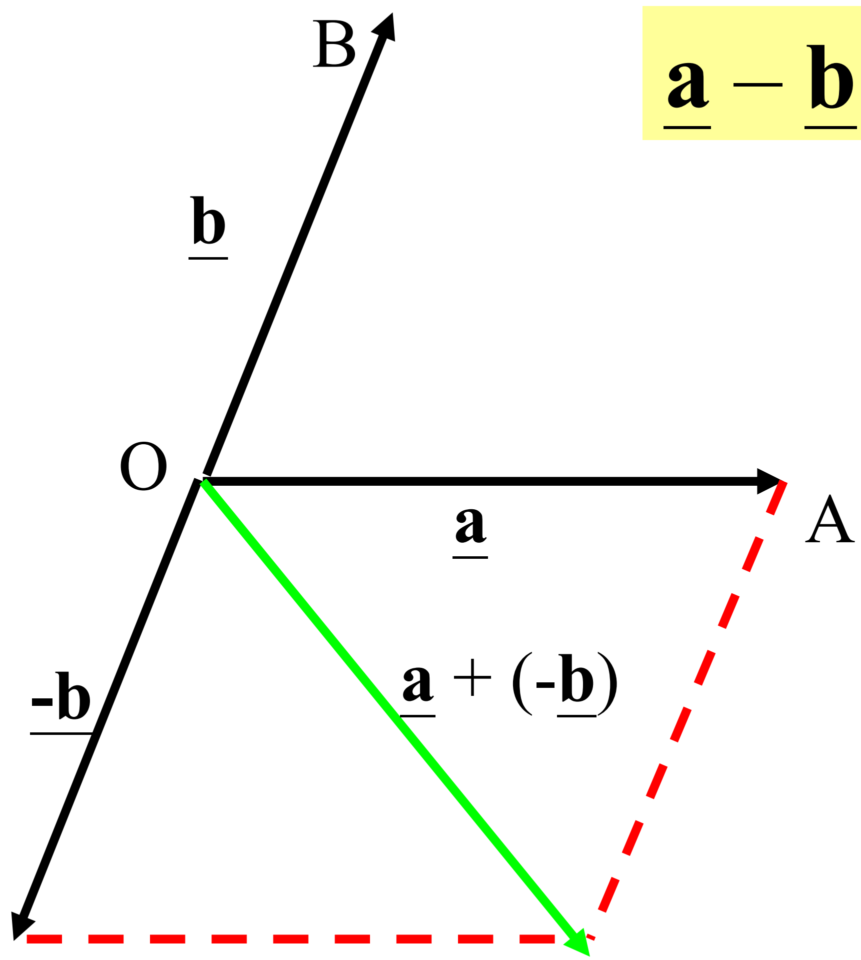
Recall: Negative of a vector: a vector having the same length (magnitude) but opposite direction



Subtraction

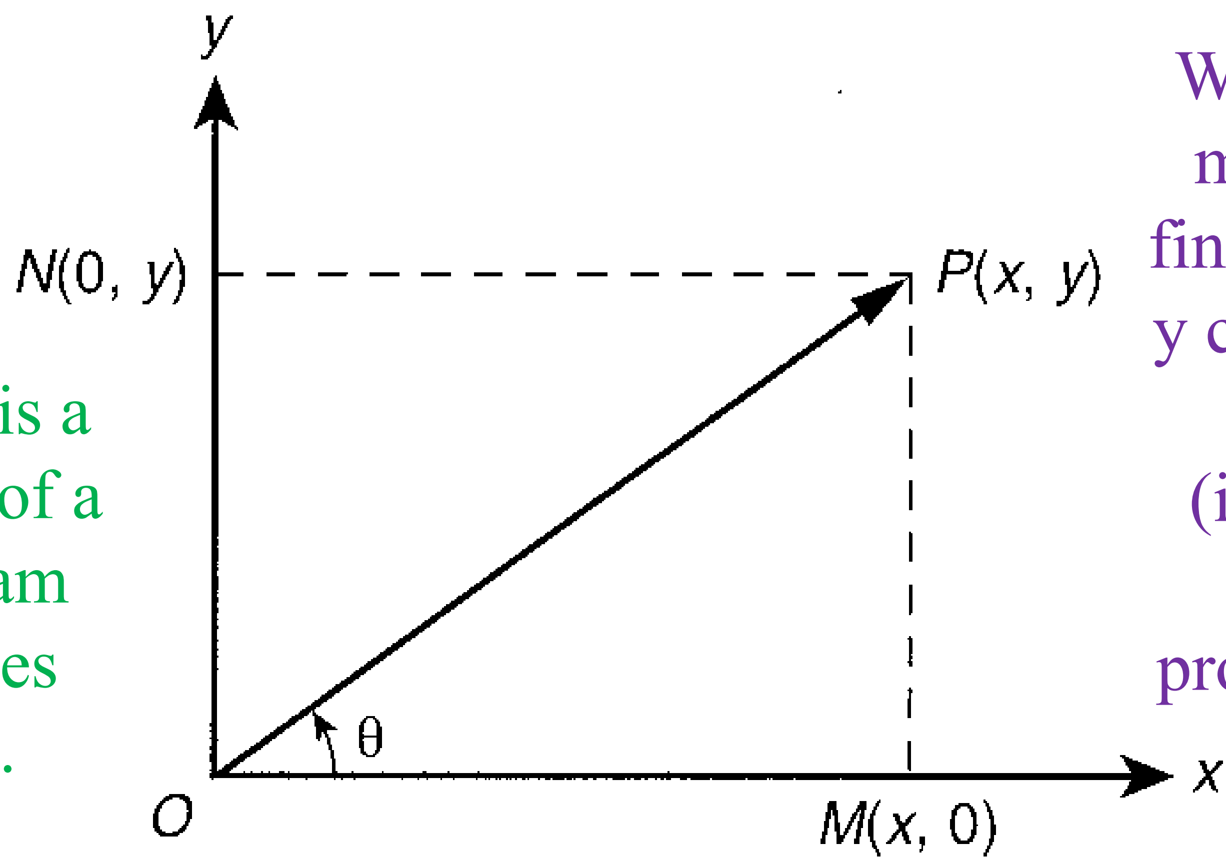
$\underline{\mathbf{a}} - \underline{\mathbf{b}} = \underline{\mathbf{a}} + (-\underline{\mathbf{b}})$

Which direction should **a-b** point?



Any vector can be broken down in components, which will be critical when adding vectors AND in the next section of class!

A rectangle is a special case of a parallelogram where angles equal 90°.



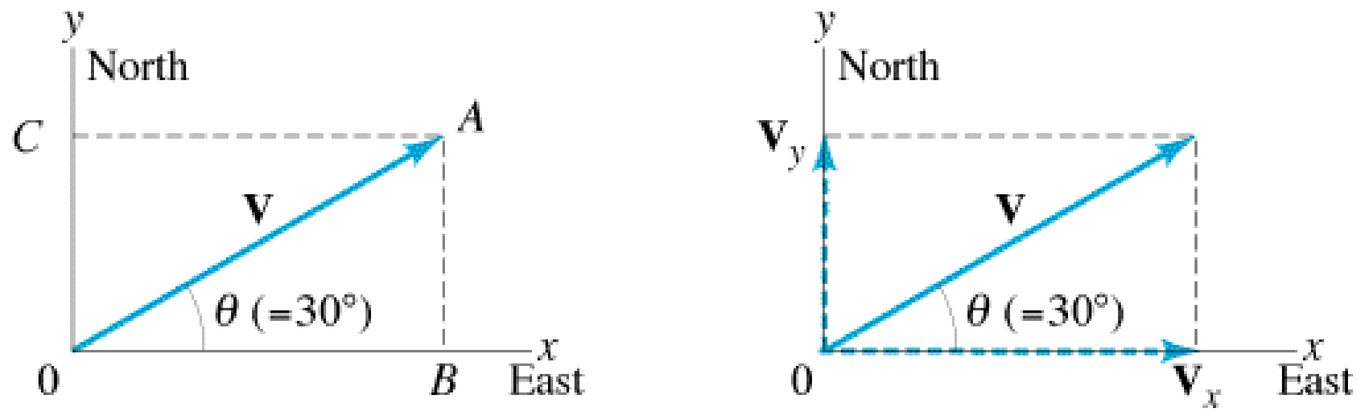
We will do this most often by finding the x and y components of velocity (important for solving problems in 2D)

OMPN is a rectangle, $\therefore\ \overrightarrow{OP} = \overrightarrow{OM} + \overrightarrow{ON}$

How to find the magnitude of OP?

Vector Arithmetic – Components

- Components of a vector (commonly velocity)



(a) (b)

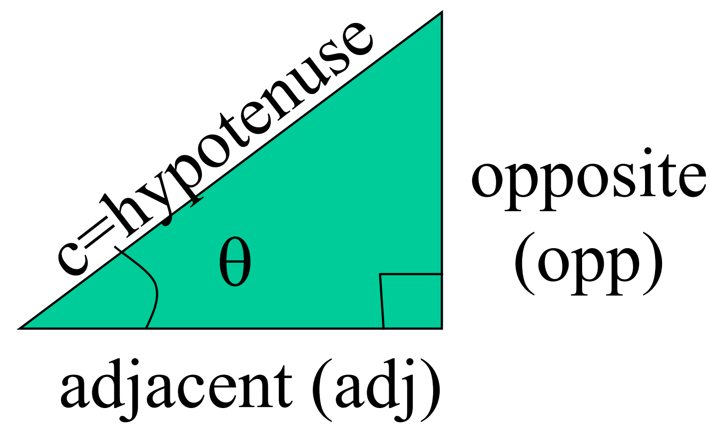
Recall for **<u>right triangles</u>**:

These are formulas with three variables. If you know 2, you can solve for the other.

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$



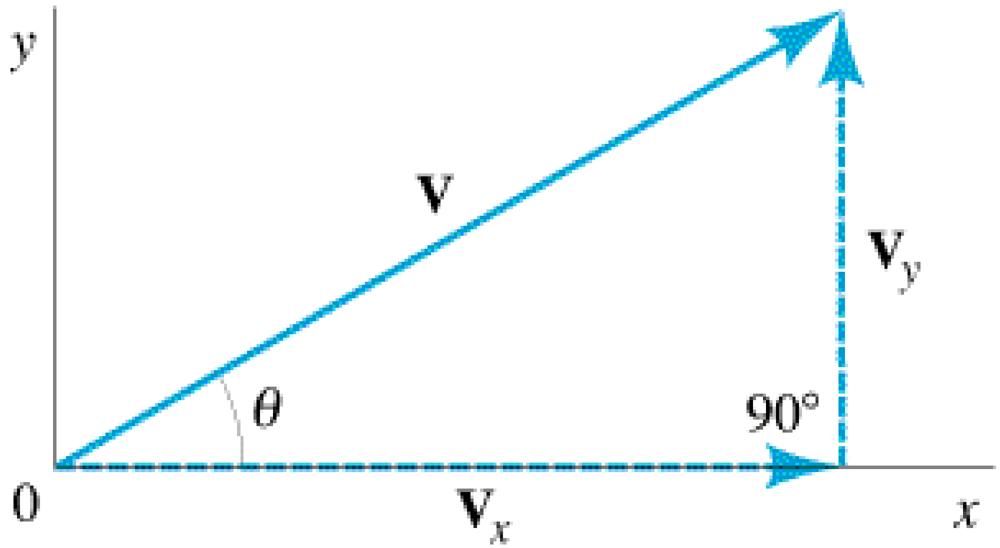
$$a^2 + b^2 = c^2$$

Pythagorean Theorem

WARNING: Make sure calculator is in degrees mode!

Only true if angle adjacent to x axis!

Otherwise, go back to definitions.



$$\sin\theta = \frac{V_y}{V}$$
$$\cos\theta = \frac{V_x}{V}$$
$$\tan\theta = \frac{V_y}{V_x}$$
$$V^2 = V_x^2 + V_y^2$$

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$$
$$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$$
$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$

$$V_x = V\cos\theta$$
$$V_y = V\sin\theta$$

(Useful if V and θ are known)

These switch if angle defined from y axis.

$$V = \sqrt{V_x^2 + V_y^2}$$
$$\tan\theta = \frac{V_y}{V_x}$$

(Useful if components are known)

V_x and V_y switch if angle defined from y axis.

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$

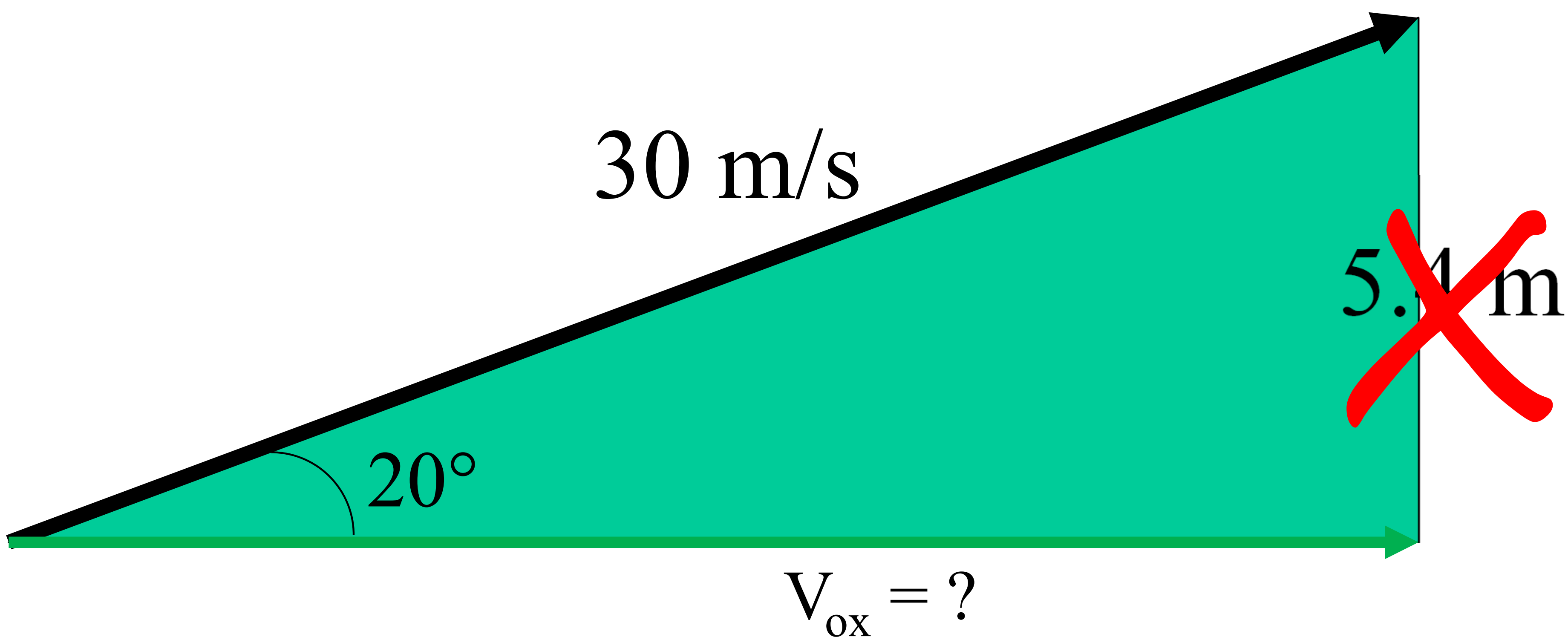
Diandra kicks a soccer ball to a max height of 5.4 m at a 20° angle from the ground with a speed of 30 m/s. What is the x (horizontal) component of the initial velocity of the soccer ball?

Hint: Draw your vector right triangle. What are the sides?

Compare your triangle with your neighbor.

Common problem:

Make sure your triangle sides all have the same units!

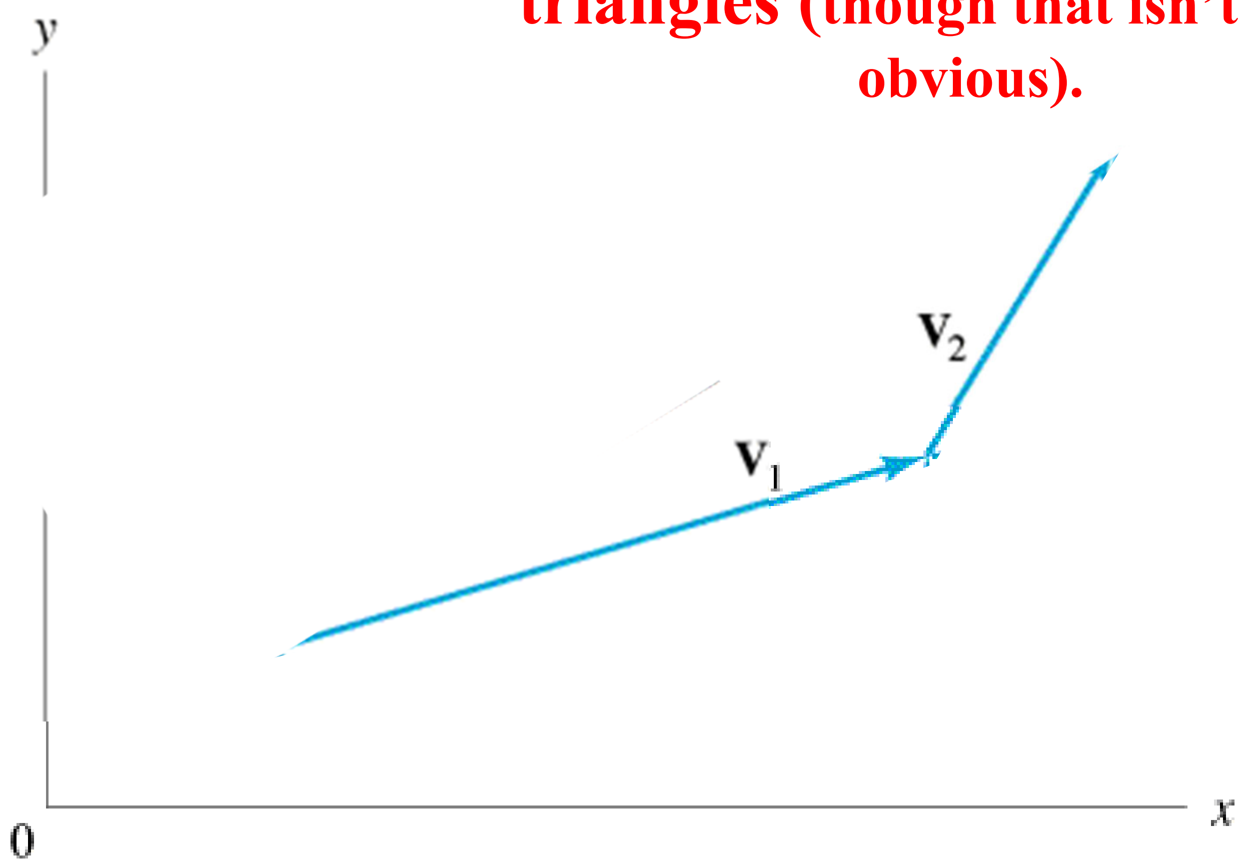


Vector Arithmetic – Components

(Will be important in Chapter 4)

Addition: $\vec{V} = \vec{V}_1 + \vec{V}_2$

In this class, we only deal with right triangles (though that isn't always obvious).



$$V_x = V_{1x} + V_{2x}$$

$$V_y = V_{1y} + V_{2y}$$

- When adding vectors, components are added separately

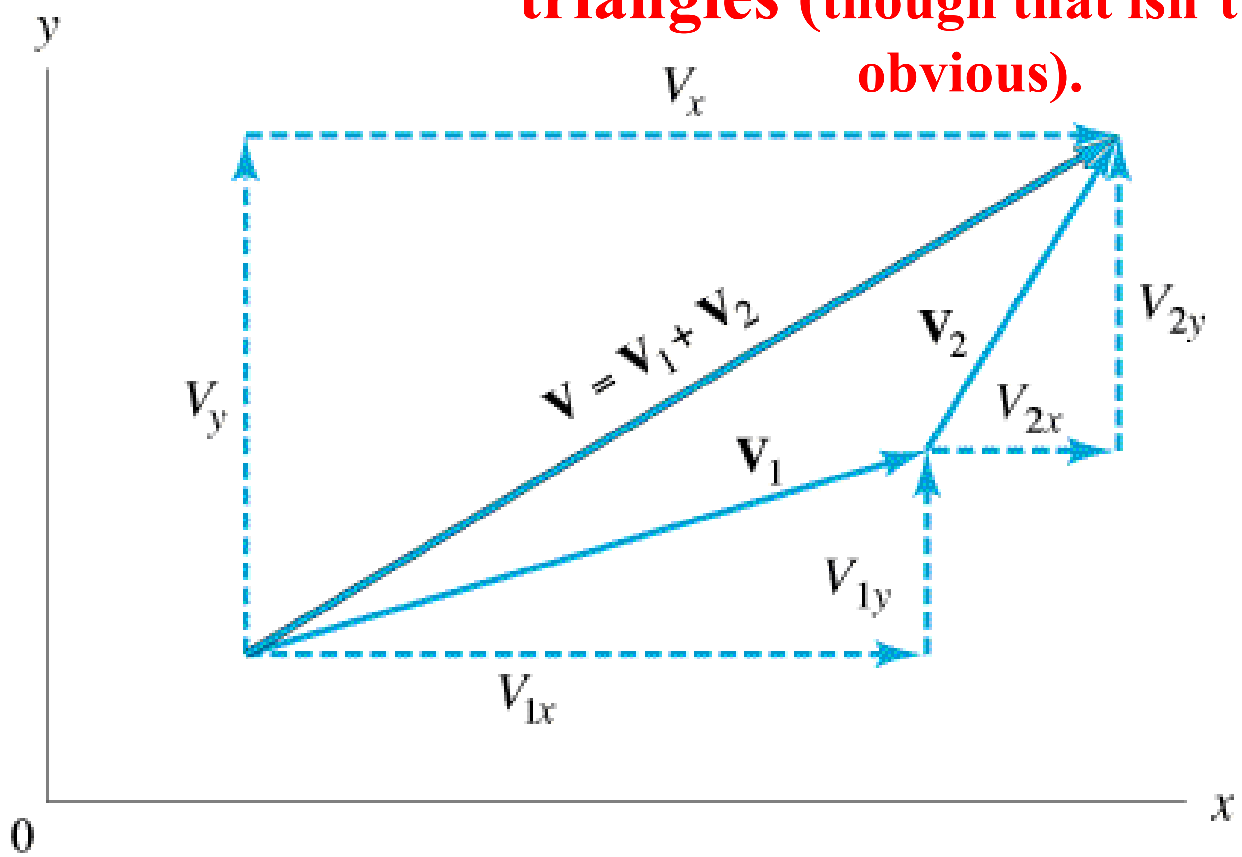
Vector Arithmetic – Components

(Will be important in Chapter 4)

Addition: $\vec{V} = \vec{V}_1 + \vec{V}_2$

In this class, we only deal with right triangles (though that isn't always obvious).

What if v_2 pointed left?



$$V_x = V_{1x} + V_{2x}$$

$$V_y = V_{1y} + V_{2y}$$

- When adding vectors, components are added separately
- Never add magnitudes of vectors

The total amount that you go East, is the amount you go East on Day 1 plus the amount that you go East on Day 2.

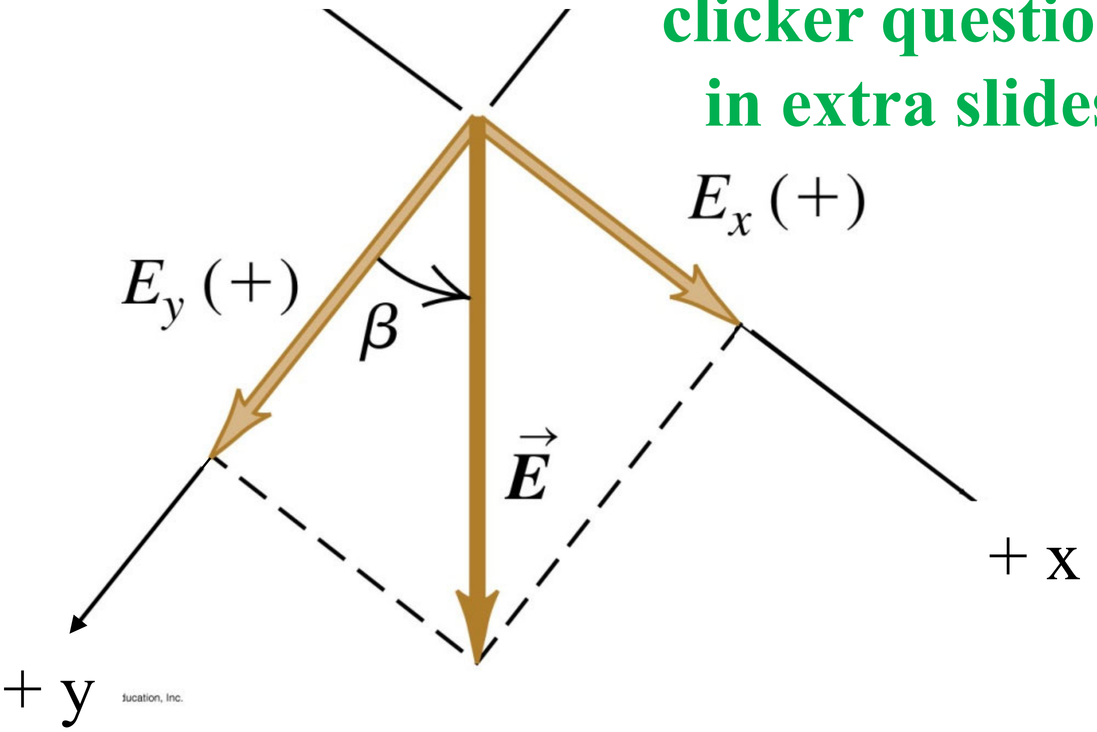
What would I do if I backtracked some?

A hiker goes on a 2-day hike. On the first day, the hiker travels 25 km Southeast. On day 2, the hiker travels 30 km East. Find the total displacement (magnitude and direction) from the point of origin.

Note the x and y axes point downward.

A few extra clicker questions in extra slides

What are the *x*- and *y*-components of the vector $\vec{E}$?



A. $E_x = E \cos \beta$, $E_y = E \sin \beta$

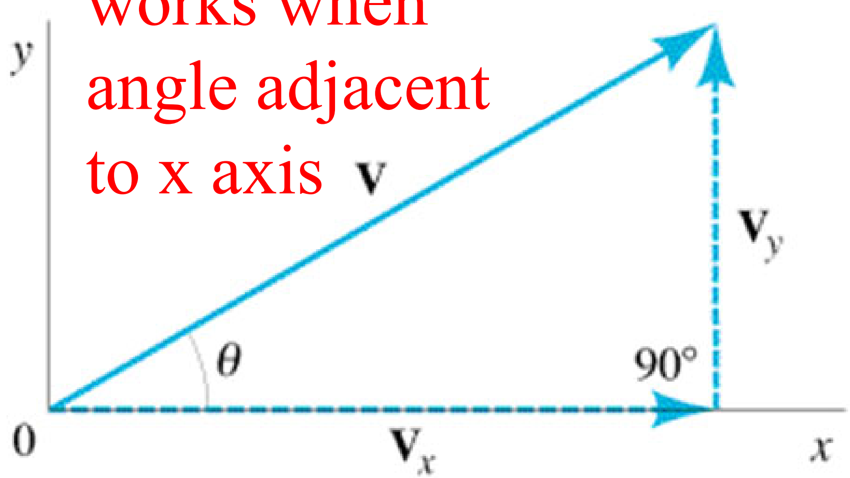
B. $E_x = E \sin \beta$, $E_y = E \cos \beta$

C. $E_x = -E \cos \beta$, $E_y = -E \sin \beta$

D. $E_x = -E \sin \beta$, $E_y = -E \cos \beta$

E. $E_x = -E \cos \beta$, $E_y = E \sin \beta$

Below only works when angle adjacent to x axis



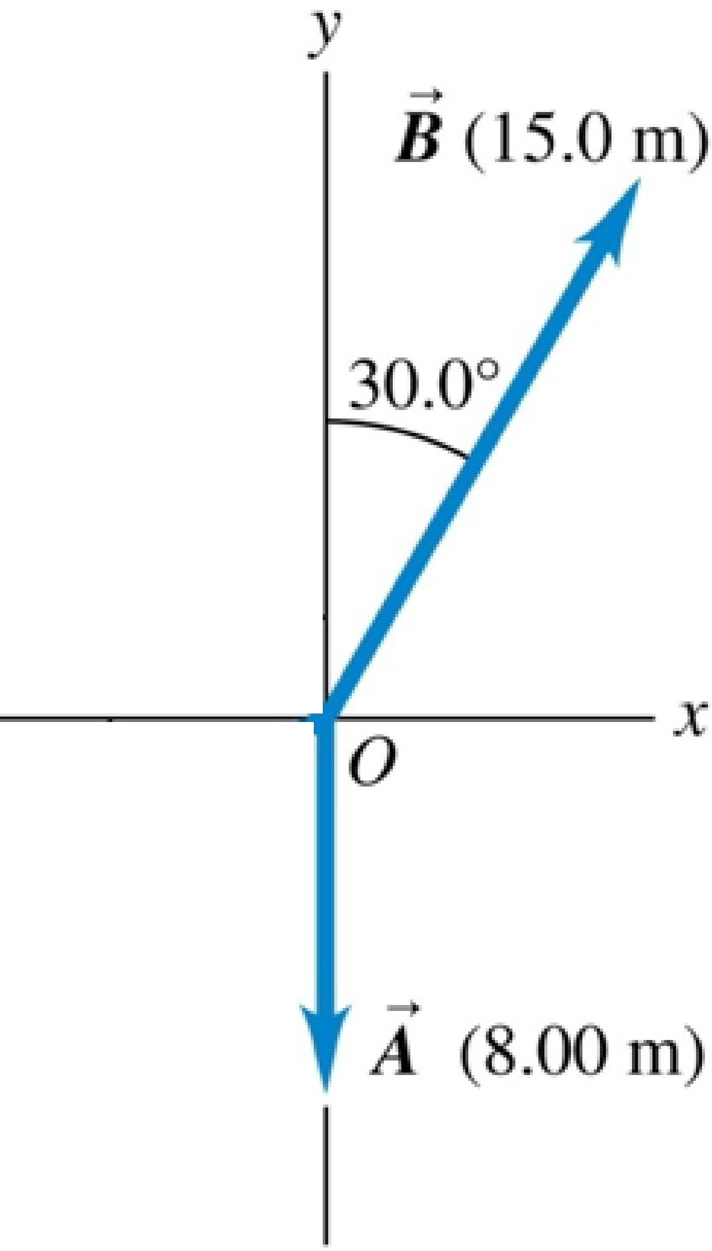
$$\sin\theta = \frac{V_y}{V}$$

$$\cos\theta = \frac{V_x}{V}$$

$$\tan\theta = \frac{V_y}{V_x}$$

$$V^2 = V_x^2 + V_y^2$$

Q18



Which is a correct statement about $\vec{A} - \vec{B}$?

A. x-component > 0, y-component > 0

B. x-component > 0, y-component < 0

C. x-component < 0, y-component > 0

D. x-component < 0, y-component < 0

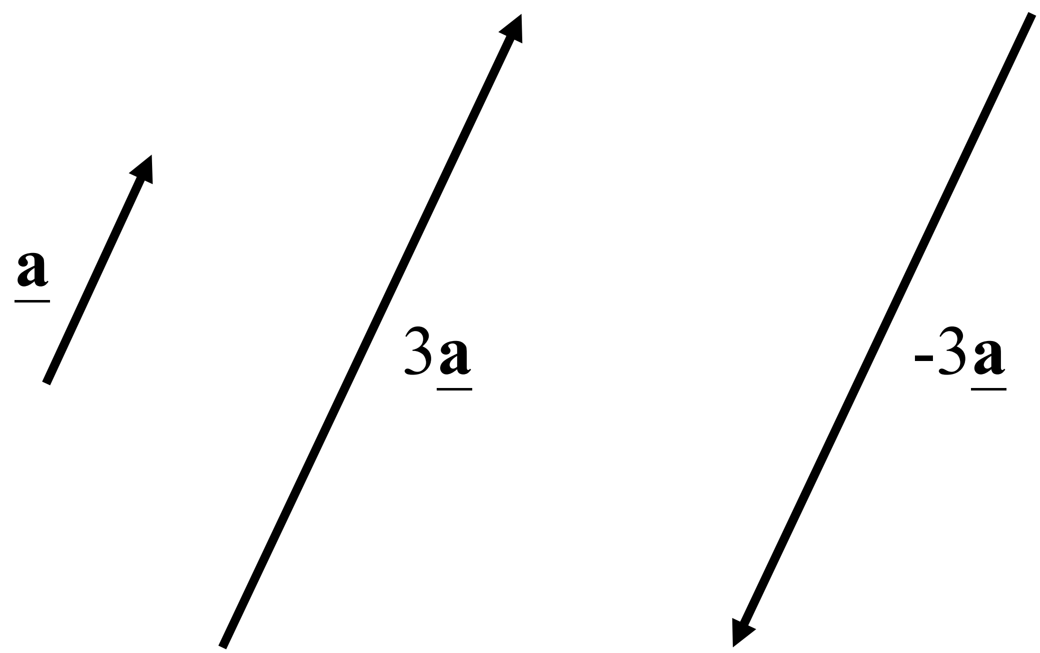
E. x-component = 0, y-component > 0

Q19

Scalar Multiplication

The product of a vector **a** and a scalar k is a vector, denoted by k**a**. This operation is called scalar multiplication.

If $k = 3$.

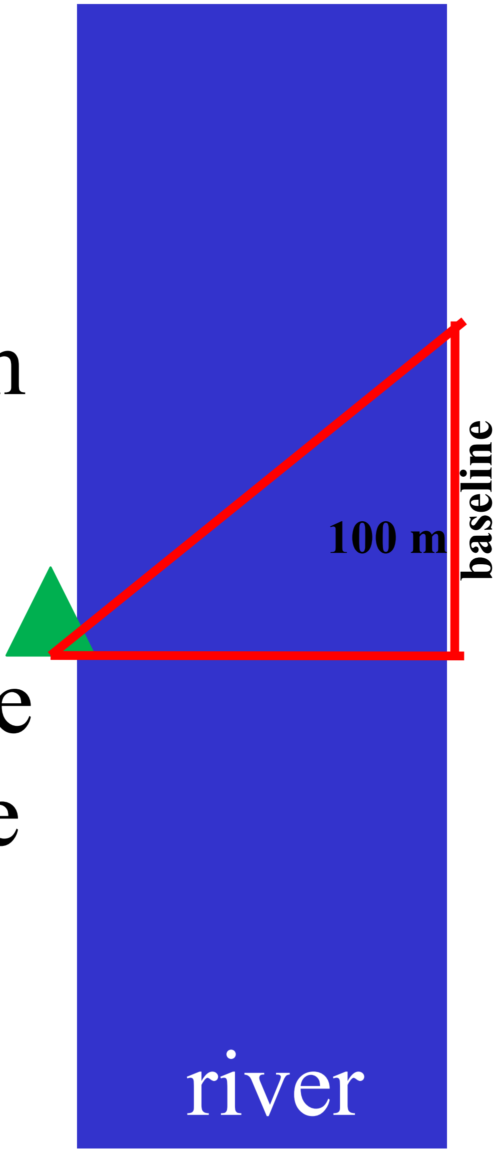


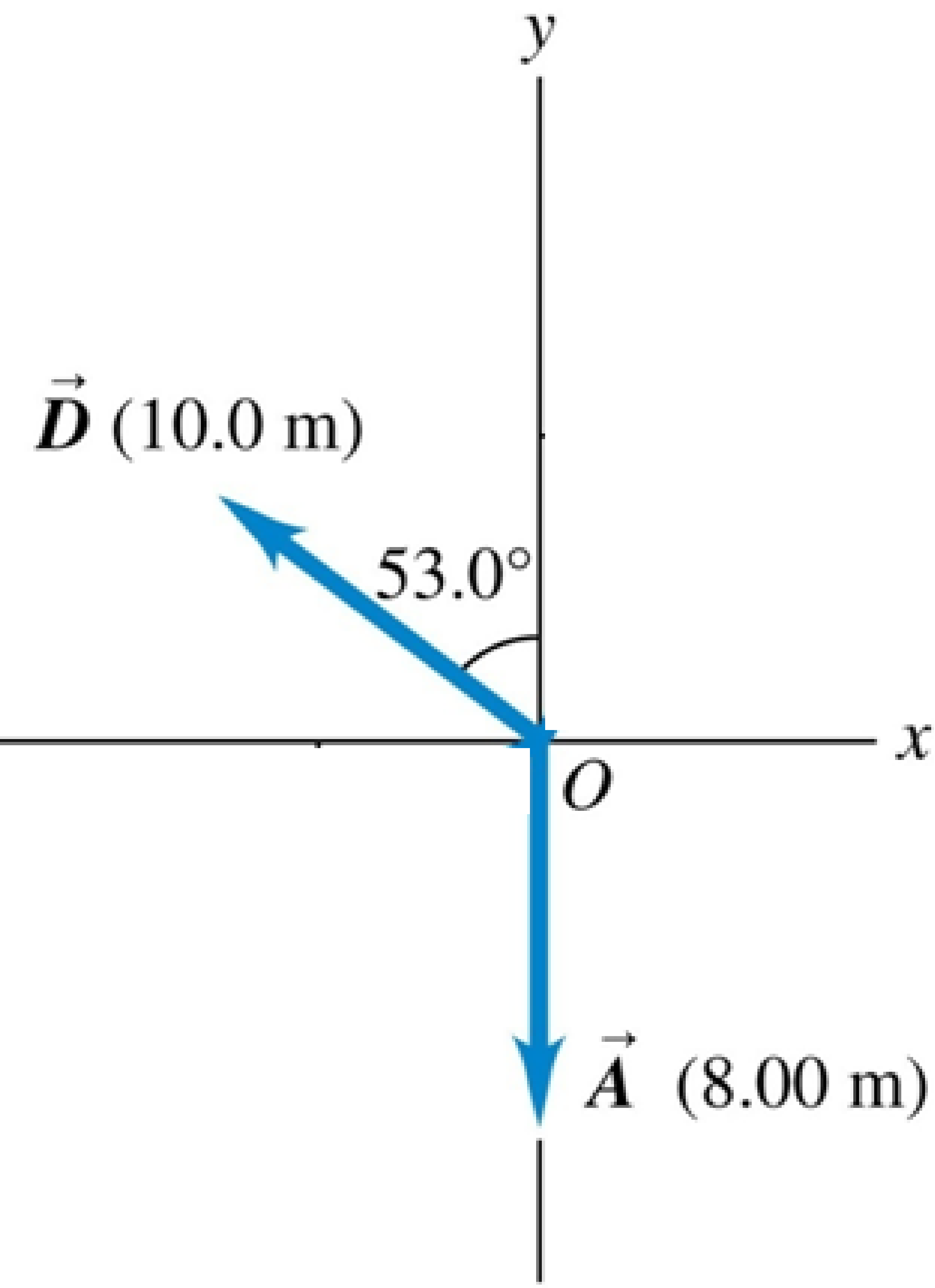
Unless k is negative, then reverses direction.

Example: Surveying the River

A surveyor wants to measure the distance across a river. Starting directly across from a big tree on the opposite bank, he walks 100 m along the riverbank to establish a baseline. Then, he sights across to the same big tree. The angle from his baseline to the tree is 35 degrees. How wide is the river?

Draw a picture.





What are the components of the vector

$\vec{E} = \vec{A} + \vec{D}$?

A. E_x = -8.00 m, E_y = -2.00 m

B. E_x = -8.00 m, E_y = +2.00 m

C. E_x = -6.00 m, E_y = 0

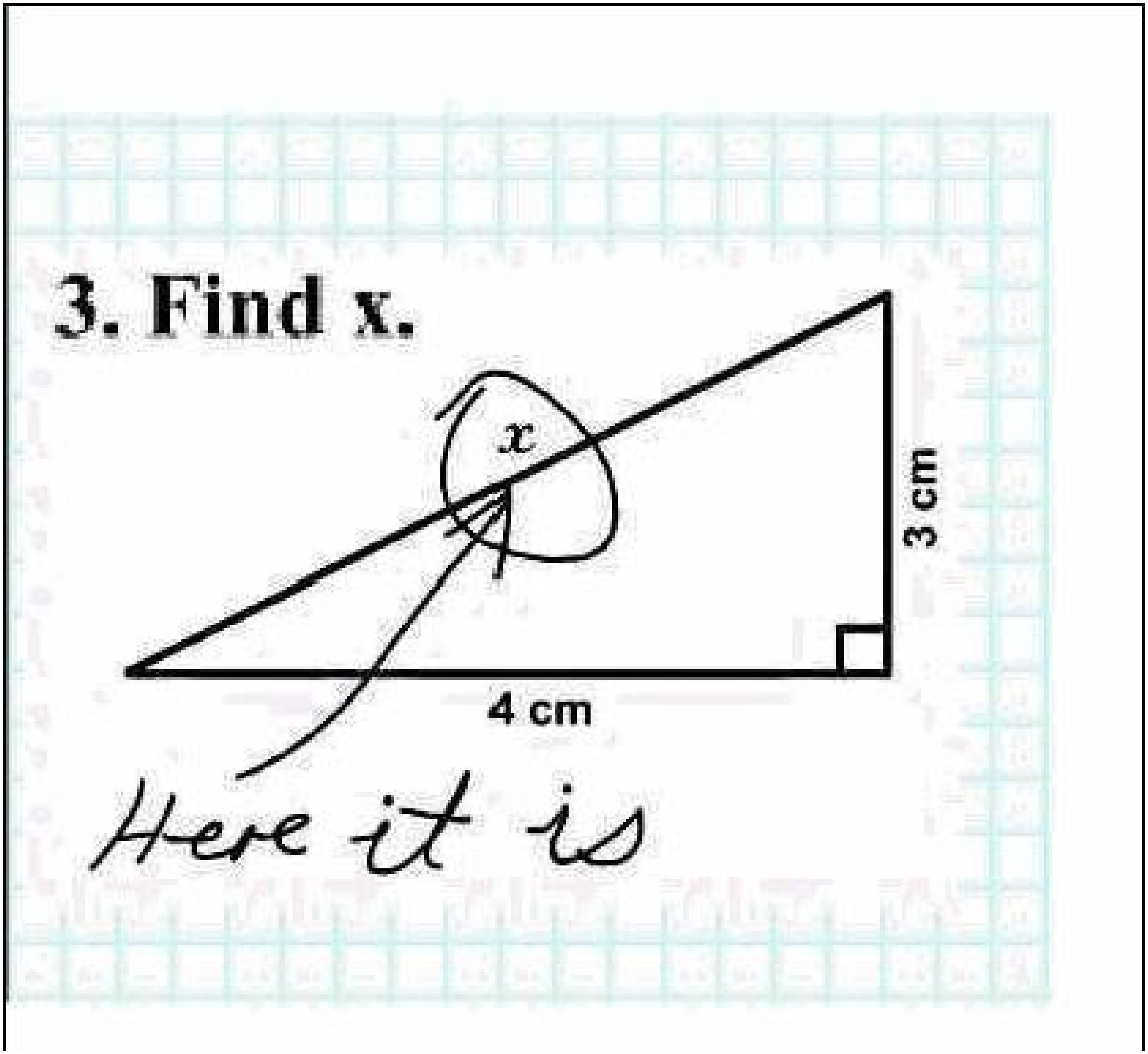
D. E_x = -6.00 m, E_y = -2.00 m

E. E_x = -10.0 m, E_y = 0

Q20

Someone actually did this!

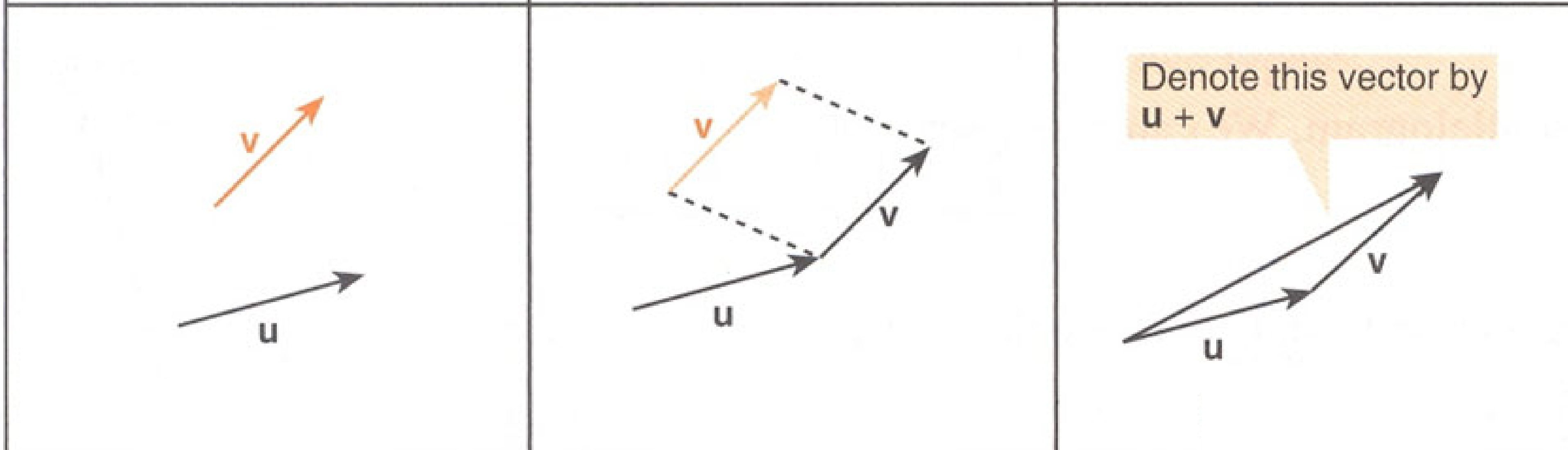
They did not get credit for it.



Basic Operation of Vectors

Important: Vectors may be moved in a coordinate system as long as magnitude and direction remain the same

Step 1: Given that **u** and **v** are two vectors on a plane.	Step 2 : Translate **v** in a parallel direction so that the initial point of **v** coincides with the terminal point of **u**.	Step 3 : A third vector, called **u** + **v**, is constructed. Its initial point coincides with that of **u** and its terminal point coincides with that of **v**.



The above procedure can be formulated as the **triangle law of addition**

Clicker Answers

Chapter/Section: Clicker #=Answer

16=B, 17=C, 18=B, 19=D, 20=A