

Next quiz Wednesday
40% Multiple choice

2 Vector/Trig Questions

Vectors are really just applying trig

3 Projectile Motion Questions
(Monday, with more practice
Wednesday)

Main Ideas in Class Today

After today, you should be able to:

- Understand vector notation
- Use basic trigonometry in order to find the x and y components of a vector

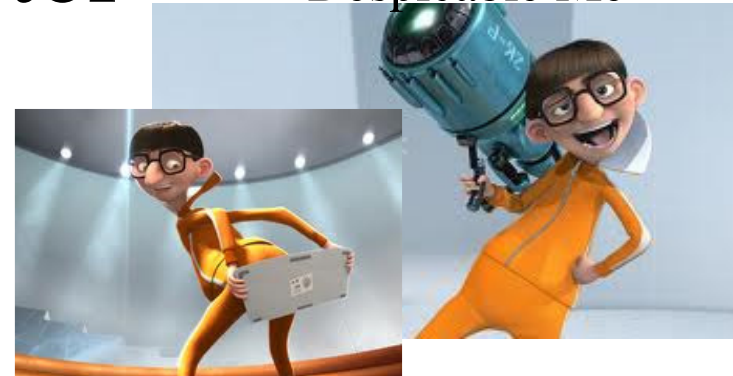
(only right triangles)

- Add and subtract vectors

Practice Problems: trig practice (1.45, 1.47, 1.49, 1.51, 1.53), vectors (1.55, 1.57, 1.61, 1.63, 1.65, 1.67, Conceptual problem 1.15)



Villian from movie
Despicable Me

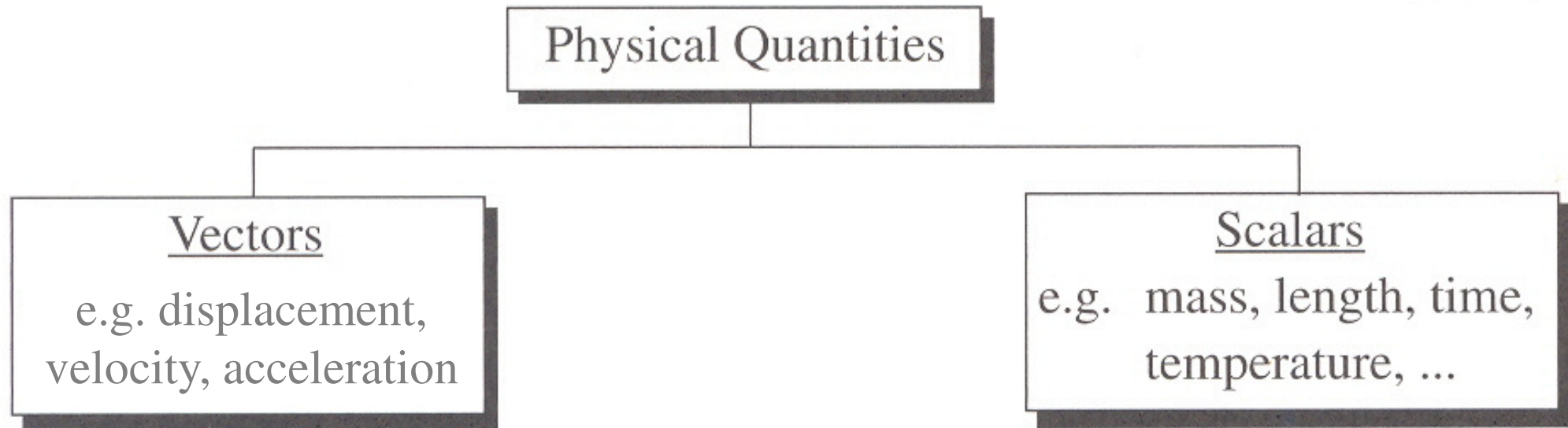


Quick Review

Quantities that are determined by a magnitude alone are called *scalars*.

Quantities that have **both** magnitude and direction are called *vectors*.

In conclusion, physical quantities can be classified into two types:



Vector Notation Varies

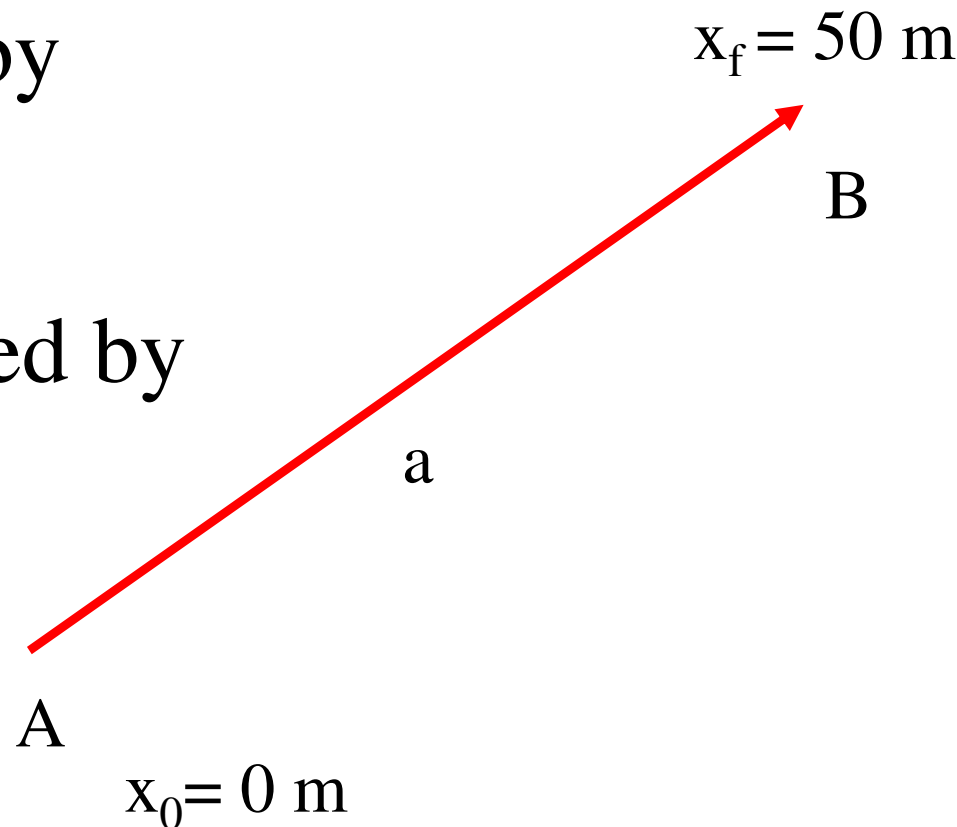
A vector may be represented by an **arrow** whose **direction** represents the **direction** of the vector and whose **length** represents the **magnitude**.

The vector may be called by

\overrightarrow{AB} , \vec{a} , \underline{a} , ***a*** (*bold*)

and its magnitude is denoted by

$|\overrightarrow{AB}|$, $|\vec{a}|$, $|\underline{a}|$, $|a|$, \boxed{a}



Two vectors are said to be *equal* ONLY if they have the *same* magnitude AND direction.

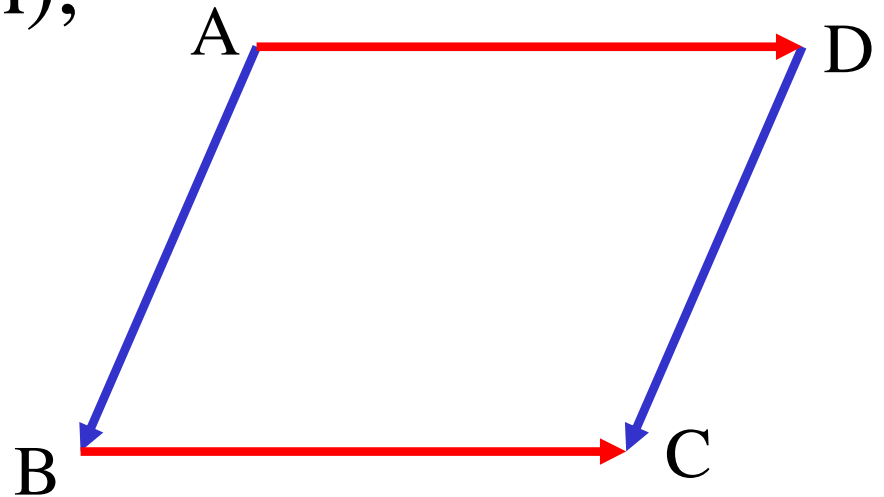
ABCD is a parallelogram (defined by a shape with 4 sides, where all opposite sides have equal length),

then

$$\vec{AB} = \vec{DC}$$

and

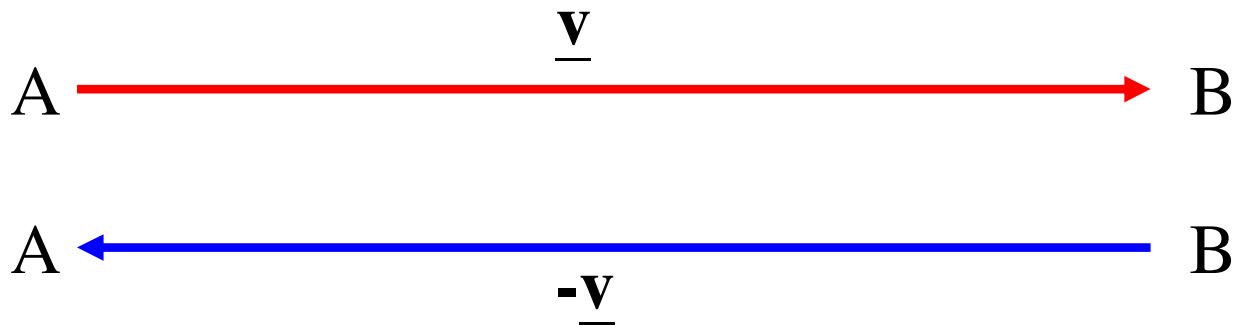
$$\vec{AD} = \vec{BC}$$



How about \vec{AB} and \vec{CD} ?

The negative vector of $\underline{\mathbf{v}}$, denoted by $-\underline{\mathbf{v}}$, is a vector having equal magnitude but opposite direction to $\underline{\mathbf{v}}$. Therefore

$$\overrightarrow{AB} = -\overrightarrow{BA}$$



Basic Operation of Vectors: Addition

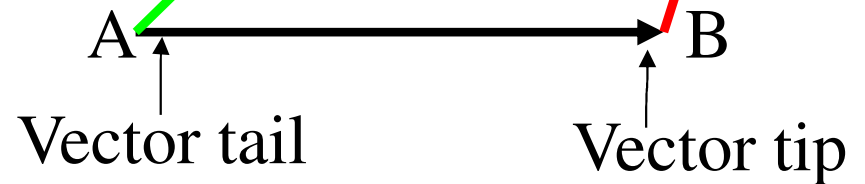


Q16

If you were to add **the red vector** to **AB** using the **triangle law** of addition, where would you put it?

triangle law of addition

$$\overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AC}$$



- a) With tail starting at A
- b) With tail starting at B
- c) Doesn't matter. In the triangle law, you would get the same answer by closing the triangle either way.

Basic Operation of Vectors

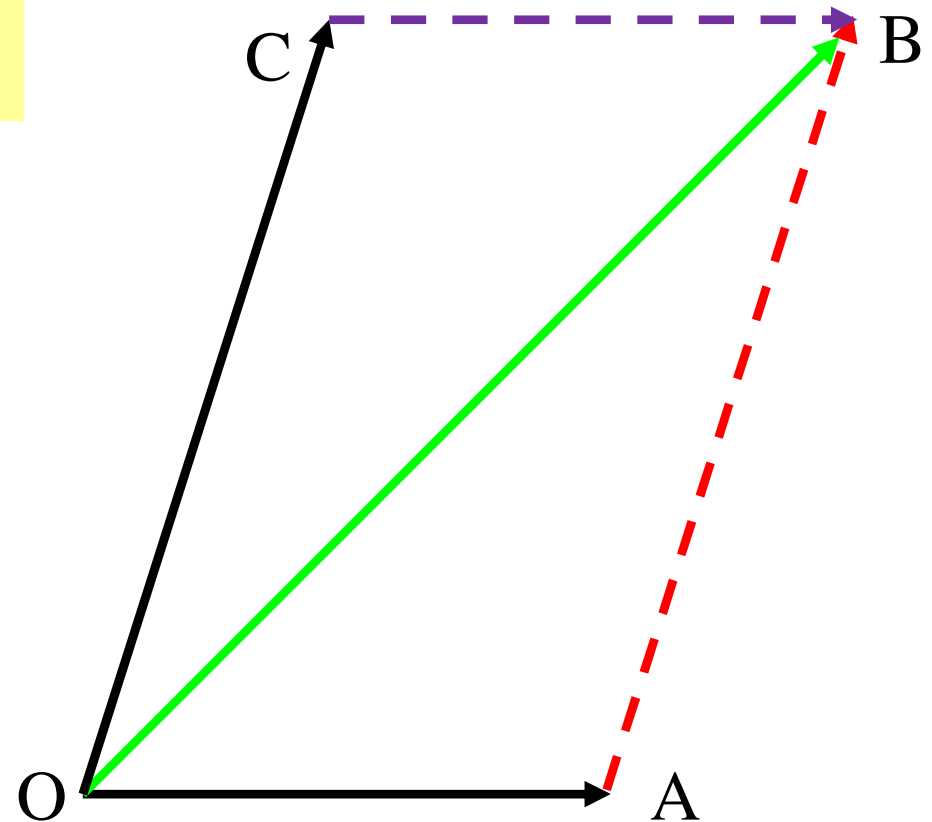
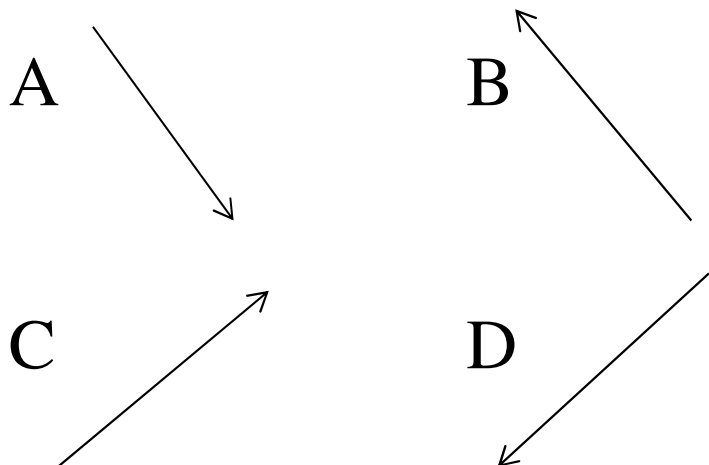


Q17

parallelogram law of addition

$$\vec{OA} + \vec{OC} = \vec{OB}$$

If you were to add these two vectors, roughly what direction would your result point?

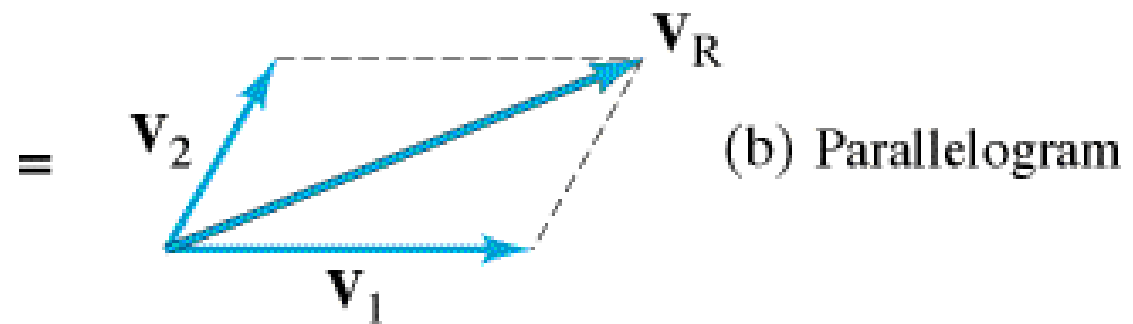
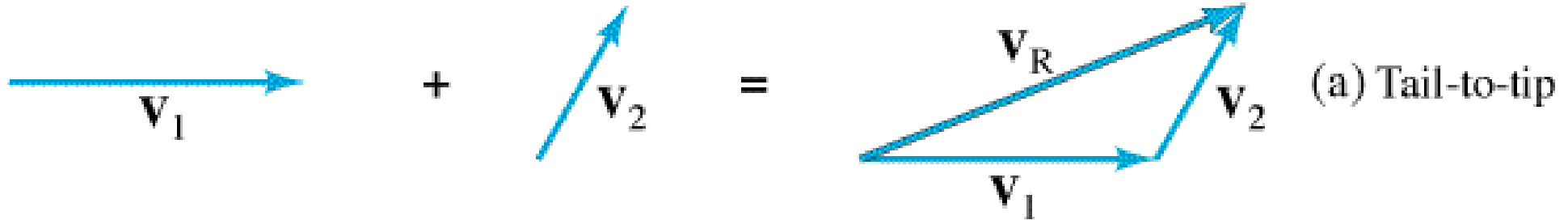


E None of the above

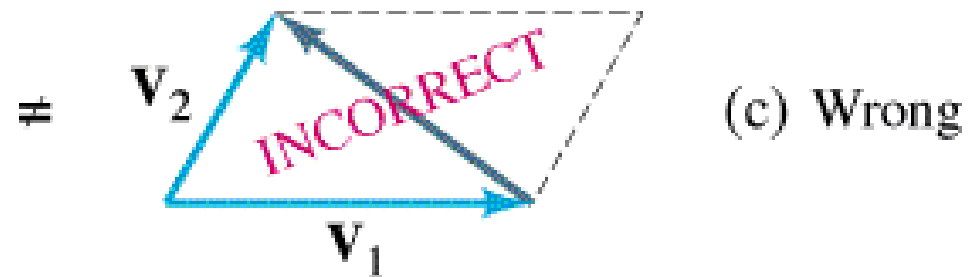
Vector Arithmetic

Addition:

$$\vec{V}_1 + \vec{V}_2 = \vec{V}_{Result}$$



It doesn't matter which order you add; the answer is the same.

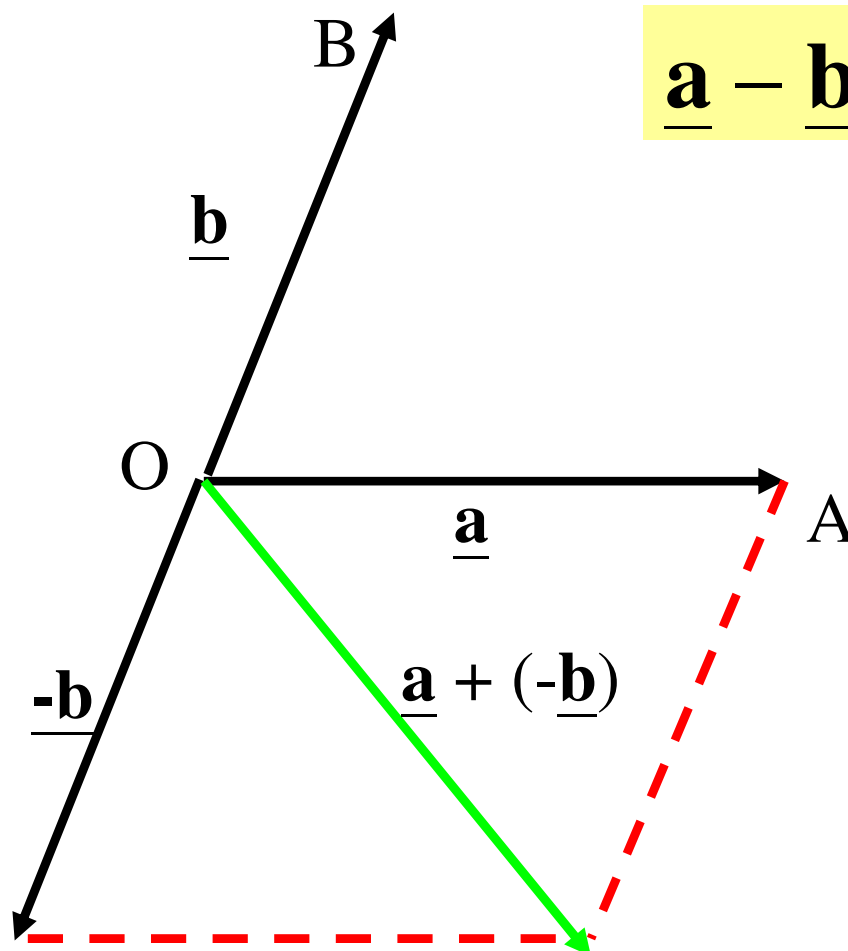


Recall: Negative of a vector: a vector having the same length (magnitude) but opposite direction

Subtraction

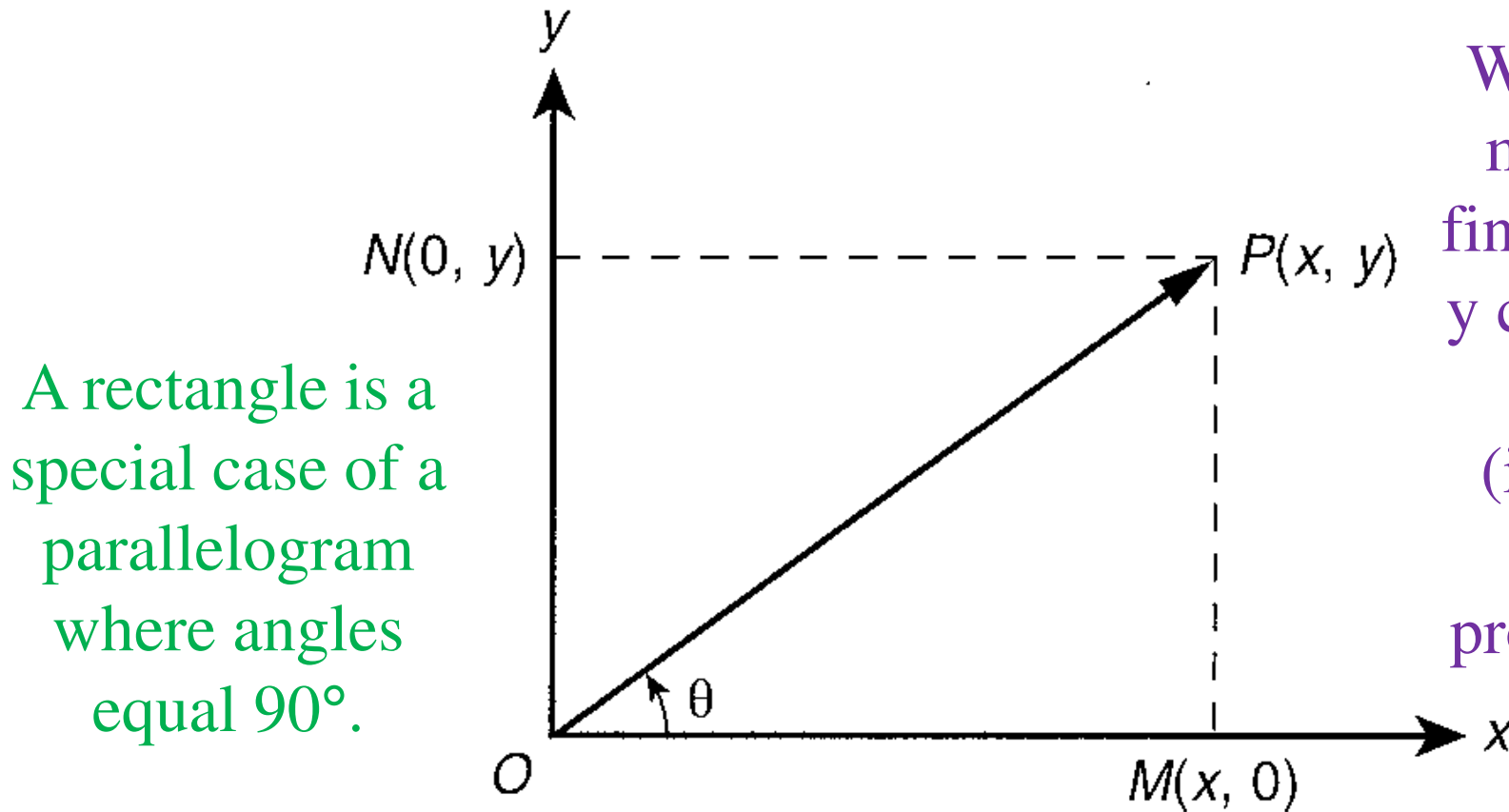


Which direction should $\underline{a} - \underline{b}$ point?



$$\underline{a} - \underline{b} = \underline{a} + (-\underline{b})$$

**Any vector can be broken down in components,
which will be critical when adding vectors!**



A rectangle is a special case of a parallelogram where angles equal 90° .

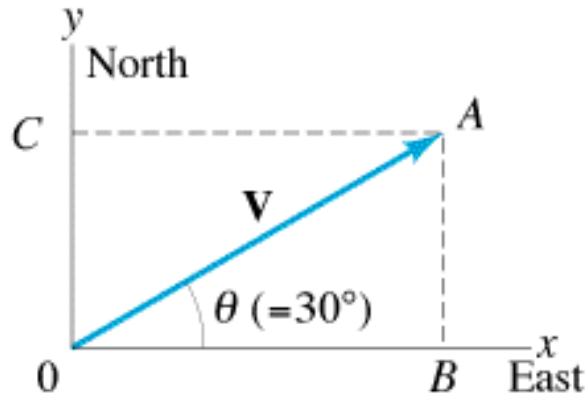
We will do this most often by finding the x and y components of velocity (important for solving problems in 2D)

OMP is a rectangle, $\therefore \vec{OP} = \vec{OM} + \vec{ON}$

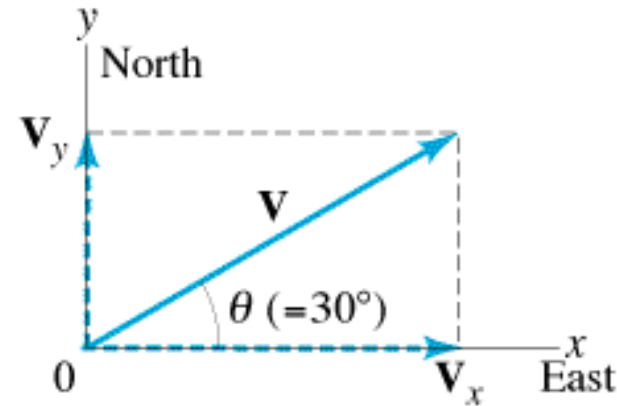
How to find the magnitude of OP?

Vector Arithmetic – Components

- Components of a vector (commonly velocity)



(a)



(b)

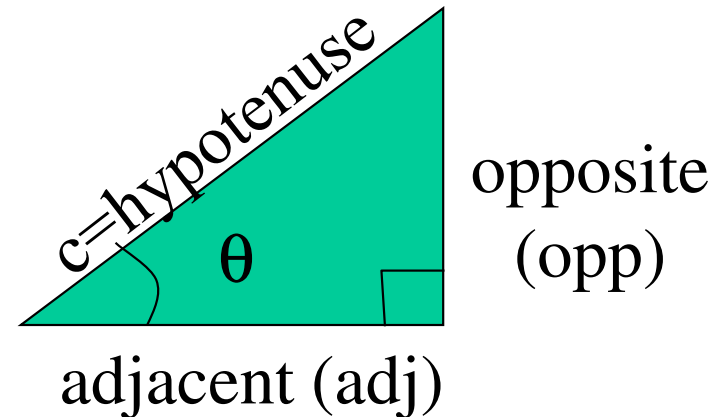
Recall for right triangles:

These are formulas with three variables. If you know 2, you can solve for the other.

$$\sin \theta = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\cos \theta = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\tan \theta = \frac{\text{opposite}}{\text{adjacent}}$$



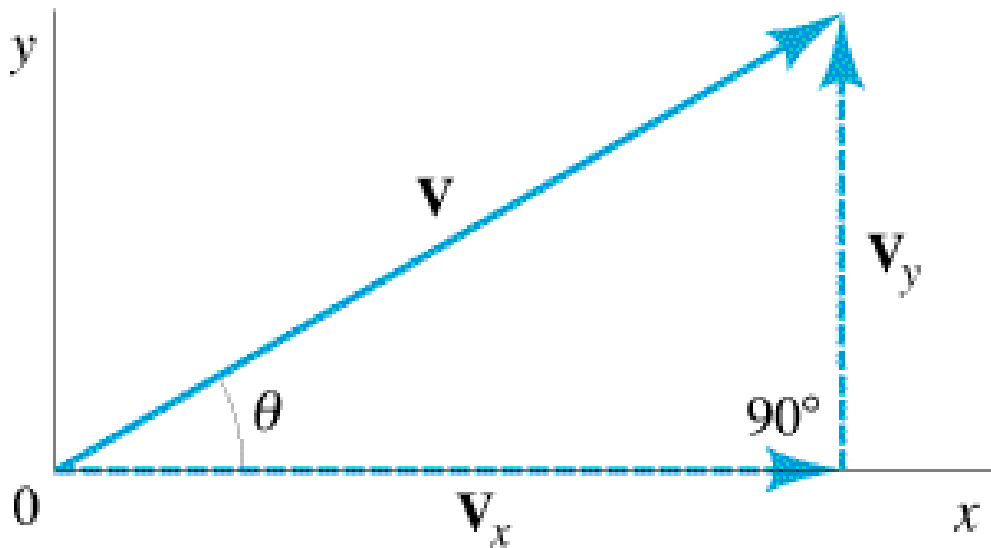
$$a^2 + b^2 = c^2$$

Pythagorean Theorem

WARNING: Make sure calculator is in degrees mode!

Only true if angle adjacent to x axis!

Otherwise, go back to definitions.



$$\sin \theta = \frac{V_y}{V}$$

$$\cos \theta = \frac{V_x}{V}$$

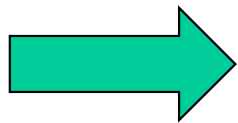
$$\tan \theta = \frac{V_y}{V_x}$$

$$V^2 = V_x^2 + V_y^2$$

$$\sin \theta = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\cos \theta = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\tan \theta = \frac{\text{opposite}}{\text{adjacent}}$$

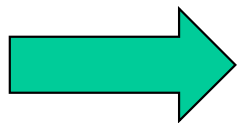


$$V_x = V \cos \theta$$

$$V_y = V \sin \theta$$

(Useful if V and θ are known)

These switch if angle defined from y axis.



$$V = \sqrt{V_x^2 + V_y^2}$$

$$\tan \theta = \frac{V_y}{V_x}$$

(Useful if components are known)

V_x and V_y switch if angle defined from y axis.



Diandra kicks a soccer ball to a max height of 5.4 m at a 20° angle from the ground with a speed of 30 m/s.

What is the **x (horizontal) component of the initial velocity** of the soccer ball?

$$\sin \theta = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\cos \theta = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\tan \theta = \frac{\text{opposite}}{\text{adjacent}}$$

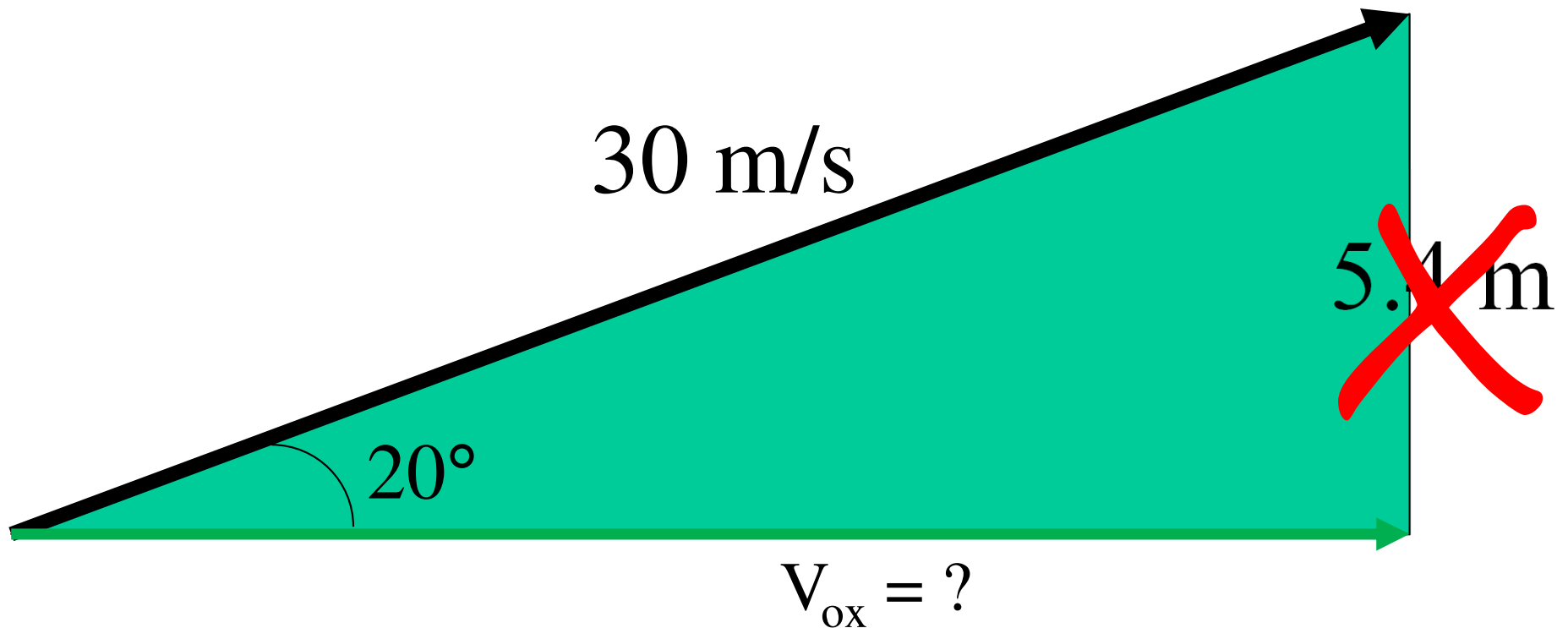
Hint: Draw your vector right triangle.

What are the sides?

Compare your triangle with your neighbor.

Common problem:

Make sure your triangle sides all have the same units!

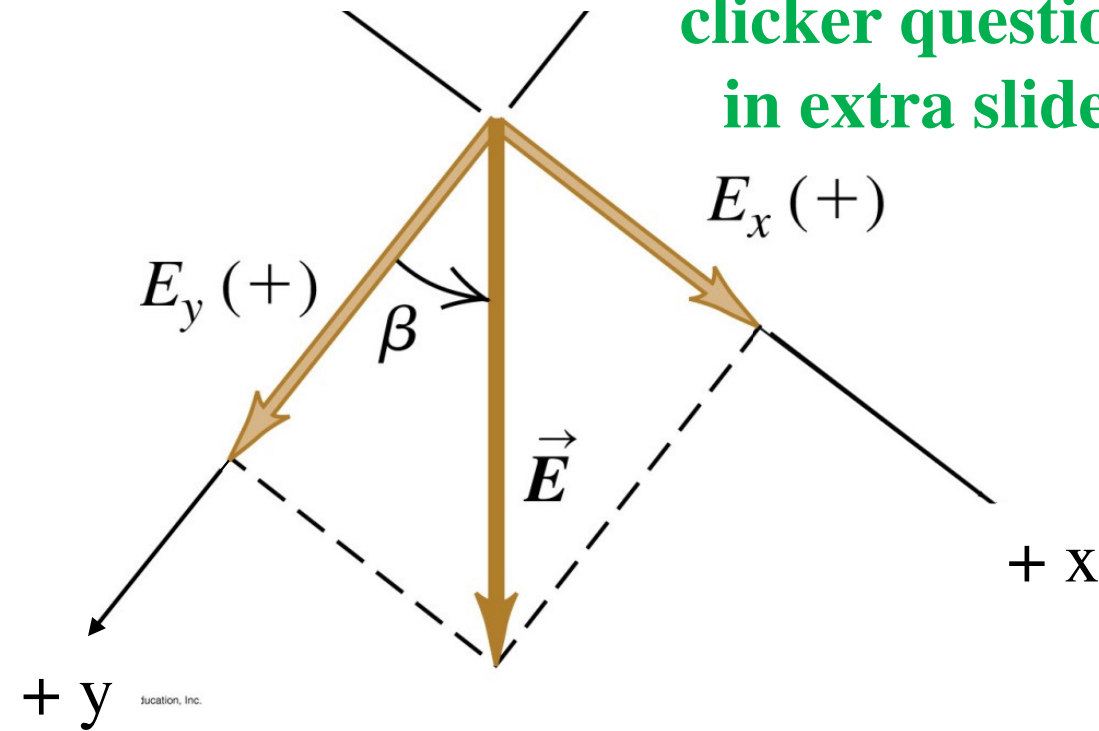




Note the x and y axes point downward.

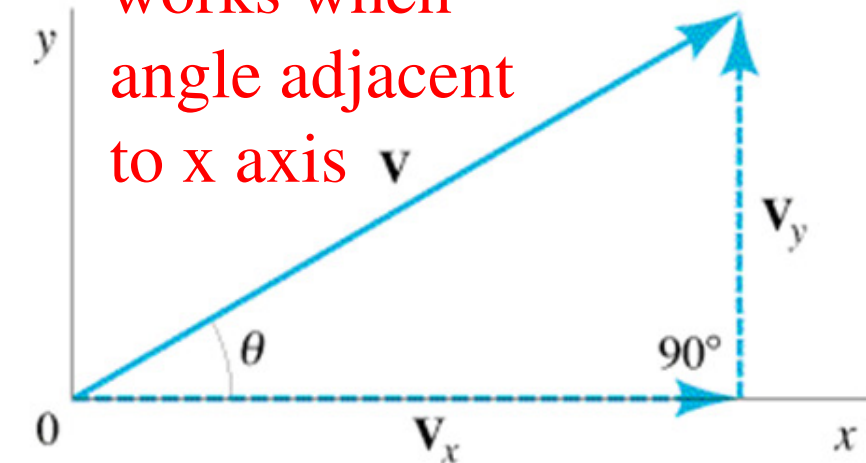
A few extra clicker questions in extra slides

What are the x- and y-components of the vector \vec{E} ?



- A. $E_x = E \cos \beta, E_y = E \sin \beta$
- B. $E_x = E \sin \beta, E_y = E \cos \beta$
- C. $E_x = -E \cos \beta, E_y = -E \sin \beta$
- D. $E_x = -E \sin \beta, E_y = -E \cos \beta$
- E. $E_x = -E \cos \beta, E_y = E \sin \beta$

Below only works when angle adjacent to x axis



$$\sin \theta = \frac{V_y}{V}$$

$$\cos \theta = \frac{V_x}{V}$$

$$\tan \theta = \frac{V_y}{V_x}$$

$$V^2 = V_x^2 + V_y^2$$



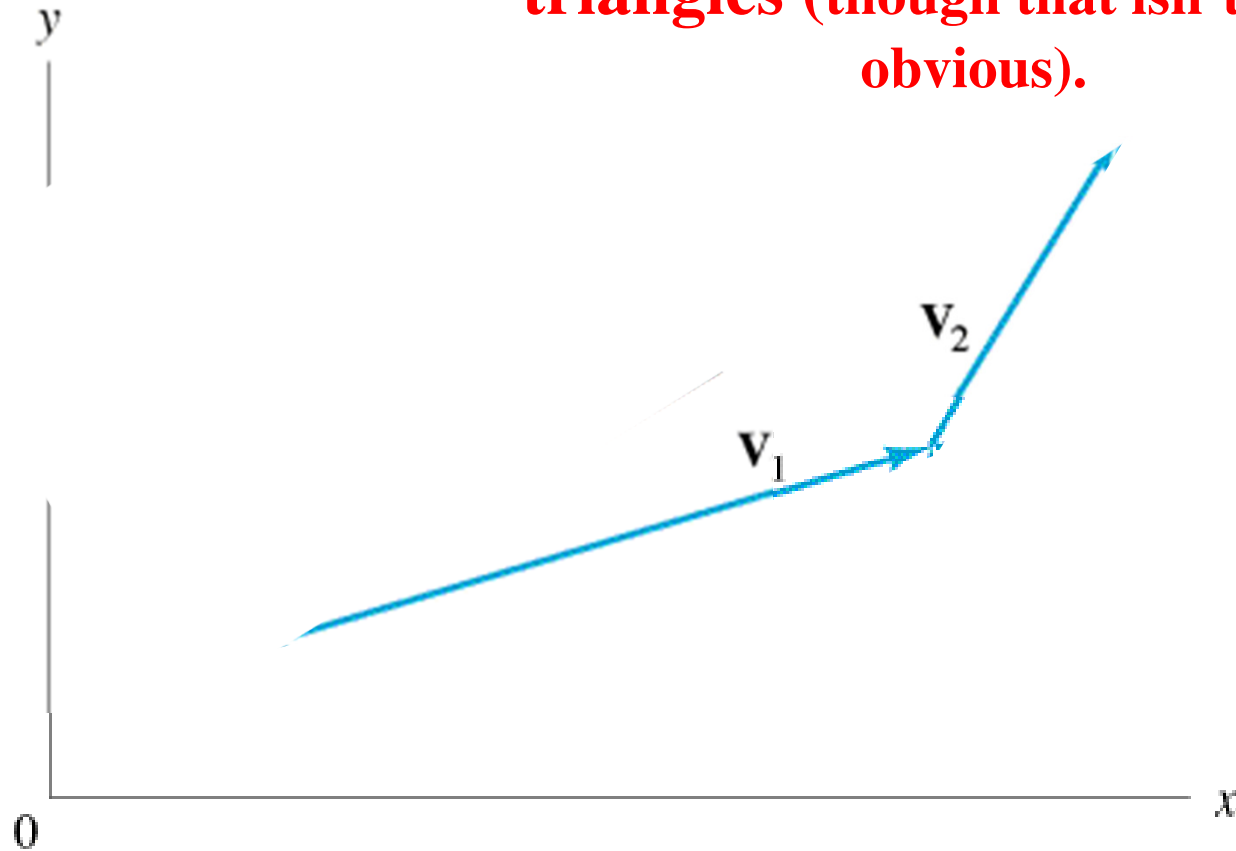
Q18

Vector Arithmetic – Components

(Will be important in Chapter 4)

Addition: $\vec{V} = \vec{V}_1 + \vec{V}_2$

In this class, we only deal with right triangles (though that isn't always obvious).



$$V_x = V_{1x} + V_{2x}$$

$$V_y = V_{1y} + V_{2y}$$

- When adding vectors, components are added separately

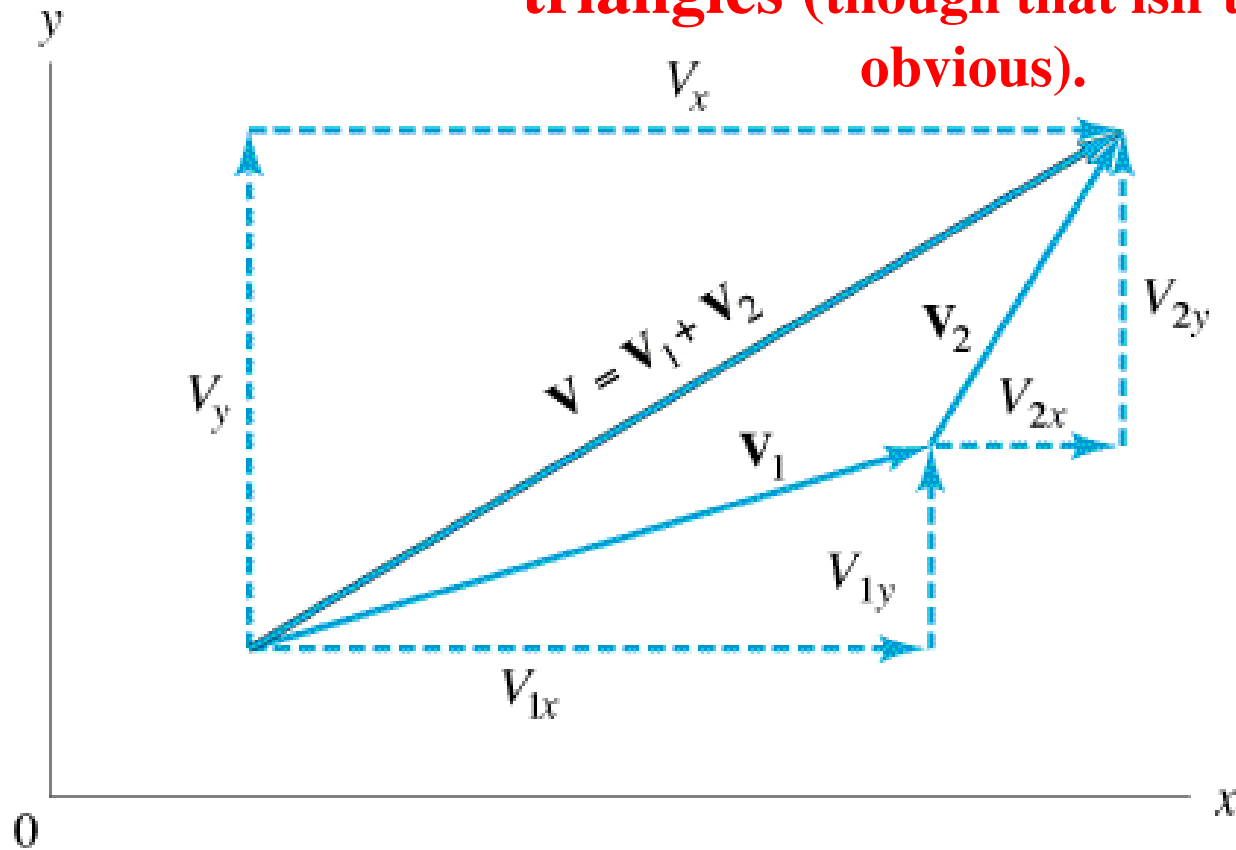
Vector Arithmetic – Components

(Will be important in Chapter 4)

Addition: $\vec{V} = \vec{V}_1 + \vec{V}_2$

In this class, we only deal with right triangles (though that isn't always obvious).

What if v_2 pointed left?



$$V_x = V_{1x} + V_{2x}$$

$$V_y = V_{1y} + V_{2y}$$

- When adding vectors, components are added separately
- **Never** add magnitudes of vectors

The total amount that you go East, is the amount you go East on Day 1 plus the amount that you go East on Day 2.



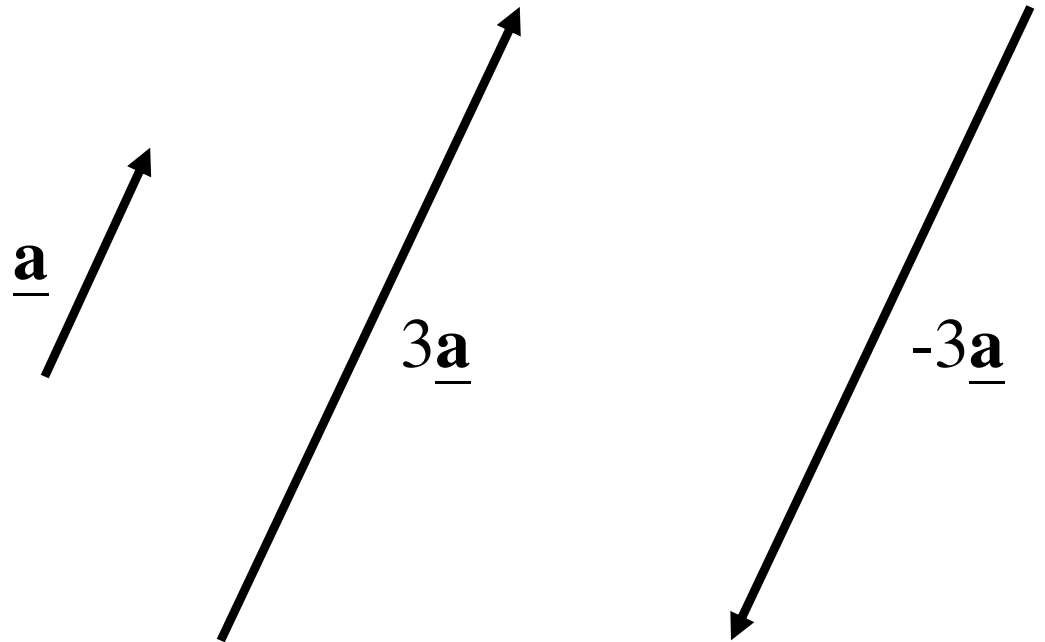
What would I do if I backtracked some?

A hiker goes on a 2-day hike. On the first day, the hiker travels 25 km Southeast. On day 2, the hiker travels 30 km East. Find the total **displacement** (magnitude and direction) from the point of origin.

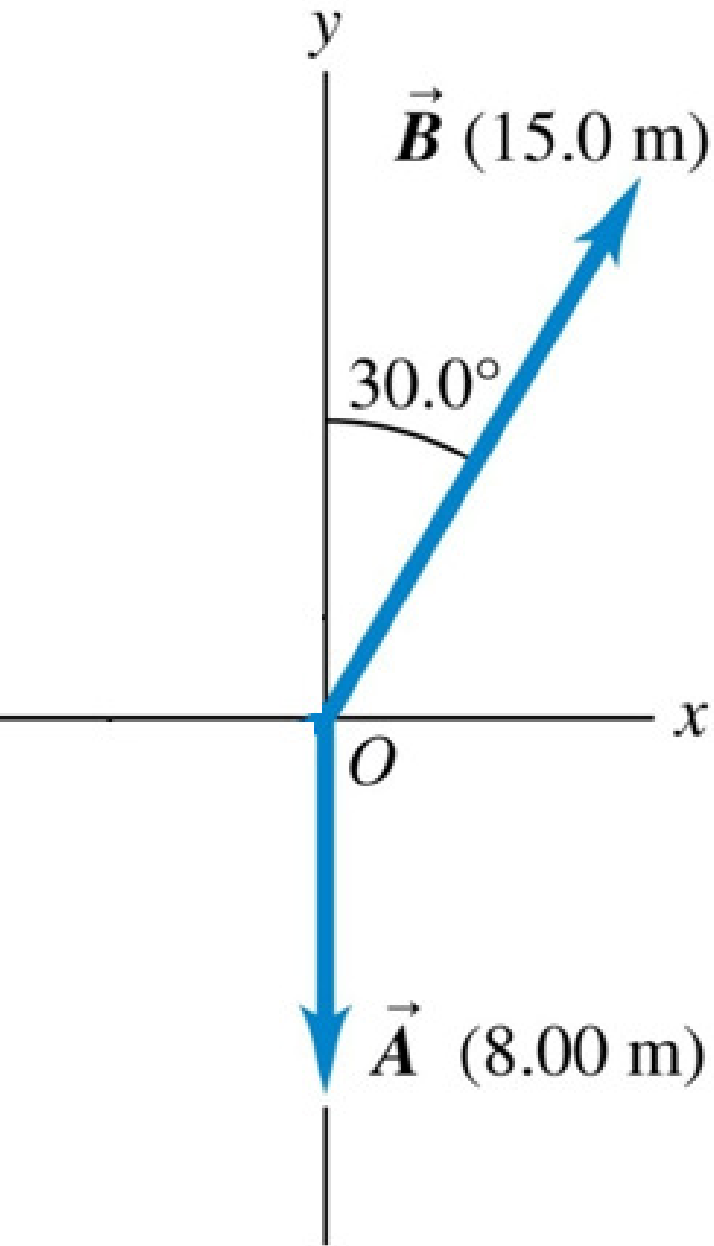
Scalar Multiplication

The product of a vector \underline{a} and a scalar k is a vector, denoted by $k\underline{a}$. This operation is called **scalar multiplication**.

If $k = 3$.



Unless k is negative, then reverses direction.



Which is a correct statement about

$$\vec{A} - \vec{B} ?$$

- A. x -component > 0 , y -component > 0
- B. x -component > 0 , y -component < 0
- C. x -component < 0 , y -component > 0
- D. x -component < 0 , y -component < 0
- E. x -component $= 0$, y -component > 0



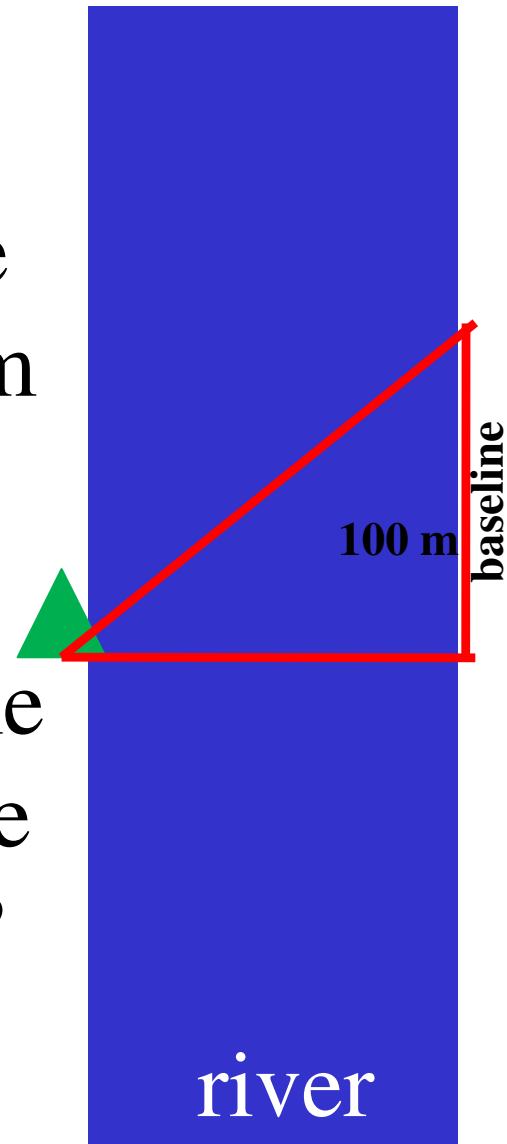
Q19



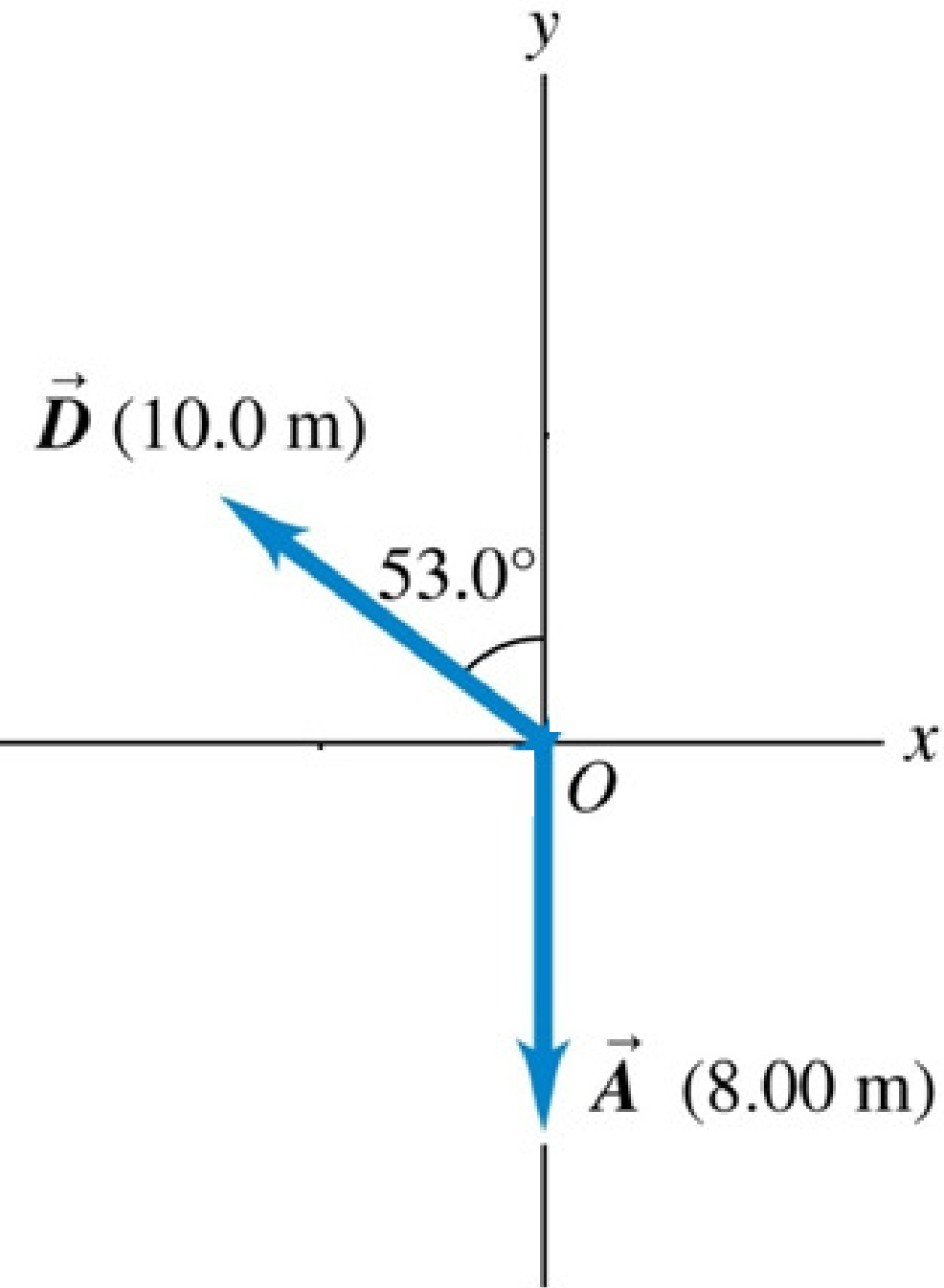
Example: Surveying the River

A surveyor wants to measure the distance across a river. Starting directly across from a big tree on the opposite bank, he walks 100 m along the riverbank to establish a baseline. Then, he sights across to the same big tree. The angle from his baseline to the tree is 35 degrees. How wide is the river?

Draw a picture.



What are the
components of the
vector
 $\vec{E} = \vec{A} + \vec{D}$?



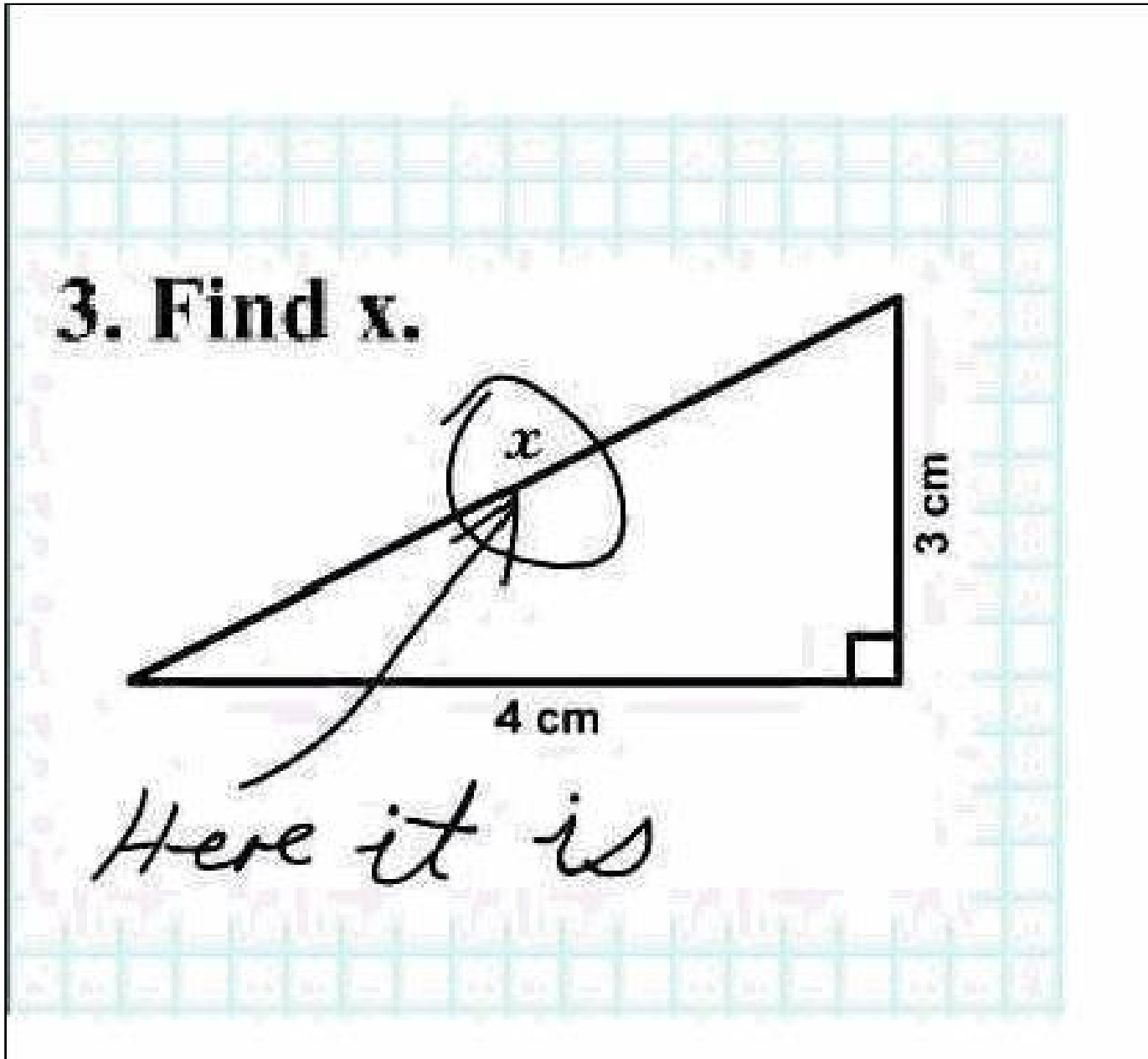
- A. $E_x = -8.00$ m, $E_y = -2.00$ m
- B. $E_x = -8.00$ m, $E_y = +2.00$ m
- C. $E_x = -6.00$ m, $E_y = 0$
- D. $E_x = -6.00$ m, $E_y = -2.00$ m
- E. $E_x = -10.0$ m, $E_y = 0$



Q20

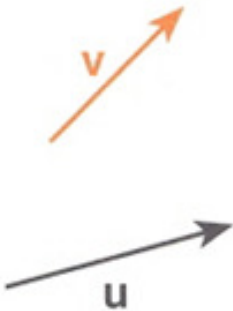
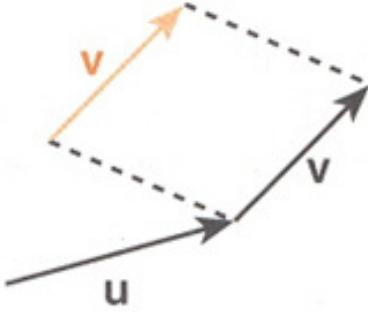
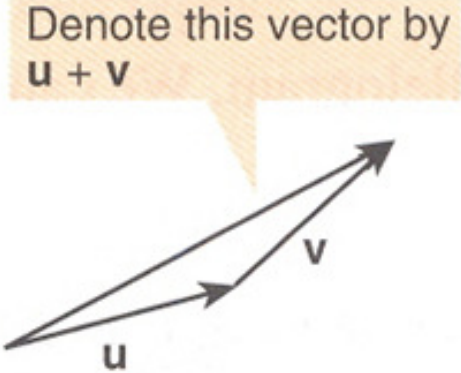
Someone actually did this!

They did not get credit for it.



Basic Operation of Vectors

Important: Vectors may be moved in a coordinate system as long as magnitude and direction remain the same

<p>Step 1: Given that \mathbf{u} and \mathbf{v} are two vectors on a plane.</p>	<p>Step 2 : Translate \mathbf{v} in a parallel direction so that the initial point of \mathbf{v} coincides with the terminal point of \mathbf{u}.</p>	<p>Step 3 : A third vector, called $\mathbf{u} + \mathbf{v}$, is constructed. Its initial point coincides with that of \mathbf{u} and its terminal point coincides with that of \mathbf{v}.</p>
		

The above procedure can be formulated as the **triangle law of addition**

Clicker Answers

Chapter/Section: Clicker #=Answer

16=B, 17=C, 18=B, 19=D, 20=A